Sierpinski Meets Pascal

We have learned all about Pascal's triangle, how to construct it and what it is used for. Now let's see if we can discover a really cool pattern behind the famous triangle.

First, use the small pink triangles in the triangle marked *Numbers* on the bulletin board to fill in each term of Pascal's triangle. Forget how it's done? Here's a reminder. Pascal's triangle starts with 1's down both sides. Then, each entry is the sum of the two entries directly above and to the left and directly above and to the right. The first two rows are done for you below as examples.

After you have completed the triangle on the bulletin board, fill in the rest of the triangle on the previous page to match it.
Now, remove all the even numbered pink triangles from the bulletin board and place them back in the large triangle on the board.

Do you notice a pattern in the remaining green triangles? You should! It’s Sierpinski’s Triangle!!

Choose one colored pencil from the triangle labeled Colored Pencils and color in the even numbers on the small triangle on the previous page. Do you see the pattern now?

Let’s try to produce the Sierpinski triangle in a different way. Do you remember learning about modular addition? This is “clock” addition. For example, three hours after 11 is not 14:00, but rather 2:00. You add three to eleven but when you get to twelve you start over again at 0. Modular addition works the same way. The addition table for mod 2 would look like this:

<table>
<thead>
<tr>
<th>Mod 2+</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Let’s use this form of addition to fill in the larger triangle on the following page. Use the same procedure as you did for filling in Pascal’s triangle. There will be 1’s down both sides and each entry will be found by adding the two entries directly above and to the left and directly above and to the right. But remember to use the Mod 2+ table! The first few rows have been done for you.
Now, use one colored pencil to color in all the 1’s.

Do you see the pattern? What do you notice about the top triangle (made of the first 8 rows) and the Sierpinski triangle you made on the bulletin board and the first page?

Use the following page to produce another Sierpinski triangle using one of the following methods:

1. Use mod 4 addition to complete the triangle. Color in all the odd numbers, and, in this case, treat zeros as if they were even numbers. Do you get the same pattern? Do you think this will work with any modular addition table?

2. Continue Pascal’s triangle and color in only the odd numbers. Do you see Sierpinski’s triangle? What do you notice about this large triangle and the large triangle you made using mod 2 addition?

When you finish please remove all of the pink triangles from the bulletin board and place them back in the triangle labeled Numbers, and return your colored pencil please!