Solving Differential Equations Numerically Using Steepest Descent

Dr. Ted Mahavier
Professor of Mathematics
Lamar University

Abstract

We demonstrate a method for solving differential equations numerically by recasting them as integral equations and then applying steepest descent to obtain a solution. For example, to solve \( y' = y \) without a boundary condition, one might let \( \phi(y) = \int_0^1 (y'(t) - y(t))^2 \, dt \) and seek out a function \( y \) so that \( \phi(y) = 0 \).

For many years it has been known that applying steepest descent based on the Euclidean norm is inefficient (disastrous) but that changing the norm on our space can result in obtaining solutions very quickly. We will give an outline of this process with lots of arm waving and simplifications while addressing the recent research by a graduate student that shows that if the perfect norm is chosen, one can expect convergence to a solution after one numerical iteration!