KUTZTOWN UNIVERSITY

Kutztown University of Pennsylvania Deptartment of Mathematics

Colloquium

3:30 p.m.

Tuesday, February 16, 2010

Lytle Hall 228

The Randy Johnsons of Abstract Algebra and Fields of Dreams

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Abstract

With the football season officially over, many anticipate the beginning of baseball. As a tribute to recently retired pitcher Randy Johnson, we will discuss big units. We are familiar with the concept of a unit: $\pm 1, \pm i$, or anything with an absolute value of 1. We will generalize this idea to define units of algebraic number fields such as $\mathbb{Q}\left(\sqrt{D}\right) = \{a + b\sqrt{D} \mid a, b \in \mathbb{Q}\}$, where $D \in \mathbb{Z}$, our first field of dreams. These units are found by solving the Pell Equation: $x^2 - Dy^2 = 1$. Surprisingly, units aren't always easy to find in these situations; they can be very big. One method that we will illustrate uses continued fraction expansions of $\sqrt{D}$. This idea will motivate us to describe an Abelian group-like algebraic structure called the *infrastructure* of the field, and will allow us to speed up the computation of these units. In the case that the units are too big to even store on a computer, we compute the natural logarithm of the unit, which is called the *regulator* of the field. Finally, we will apply these ideas to fields of rational functions modulo a prime, our second field of dreams. The benefit of these so-called *function fields* is that the regulator is now an integer and also the order of a group related to the units and infrastructure of the field. Furthermore, the infrastructure has applications to cryptography, hence our interest in the problem. All are welcome to attend; this talk will be accessible to students and faculty alike!

3:00 p.m.
refreshments served

3:30 p.m.
talk begins