

SPRING 2009
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HANDOUT § 0
INDUCTIVE REASONING EXERCISES

1. The Circle Problem

Let C be a circle with radius one centred at $(0,0)$.

Set 1. Let A_1 and A_2 be points on the circle. Call them vertices. Connect all vertices with a chord (in this case A_1 and A_2 are connected by a chord). Consider the interior of the circle. Let region be defined as an interior part of the circle such that a set of chords separates it from other interior parts of the circle. Count the number of points (2), the number of chords, (1), and the number of regions (2).

Hypothesize as to the general number of points, chords, and regions. How confident are you with your predictions? Do you think your hypotheses are true or false? Why?

Set 2. Let A_1 , A_2 , and A_3 be points on the circle. Call them vertices. Connect all vertices with a chord (in this case A_1 and A_2 are connected by a chord, A_1 and A_3 are connected by a chord, and A_2 and A_3 are connected by a chord). Consider the interior of the circle. Let region be defined as an interior part of the circle such that a set of chords separates it from other interior parts of the circle. Count the number of points (3), the number of chords, (3), and the number of regions (4).

Hypothesize as to the general number of points, chords, and regions. How confident are you with your predictions? Do you think your hypotheses are true or false? Why?

Exercise: Do this for set 3, 4, and 5. Hypothesize as to the general number of points, chords, and regions. How confident are you with your predictions? Do you think your hypotheses are true or false? Why?

2. The Polynomial Problem

Consider $f(n) = n^2 + n + 41 \quad \forall n \in \mathbb{N}$

Set 1. Consider $f(1)$. It is 43. What kind of natural number is it? Hypothesize as to the general rule for $f(n)$. How confident are you with your prediction? Do you think your hypothesis is true or false? Why?

Set 2. Consider $f(2)$. It is 47. What kind of natural number is it? Hypothesize as to the general rule for $f(n)$. How confident are you with your prediction? Do you think your hypothesis is true or false? Why?

Exercise: Do this for set 3, 4, 5, 6, 7, 8, 9, 10, and 11. Hypothesize as to the general number of points, chords, and regions. How confident are you with your predictions? Do you think your hypotheses are true or false? Why?

3. The Really Hard Polynomial Problem

Consider $f(n) = 991n^2 + 1 \quad \forall n \in \mathbb{N}$

Set 1. Consider $f(1)$. It is 992. What kind of natural number is it? Hypothesize as to the general rule for $f(n)$. How confident are you with your prediction? Do you think your hypothesis is true or false? Why?

Set 2. Consider $f(2)$. It is 3965. What kind of natural number is it? Hypothesize as to the general rule for $f(n)$. How confident are you with your prediction? Do you think your hypothesis is true or false? Why?

Exercise: Do this for set 3, 4, 5, 6, 7, 8, 9, 10, and 11. Hypothesize as to the general number of points, chords, and regions. How confident are you with your predictions? Do you think your hypotheses are true or false? Why?

Hint 1. The Circle Problem

Consider $n = 6$

Hint 2. The Polynomial Problem

Consider $f(40)$

Solution 1¹: For $n = 6$ the conjecture about regions fails for $2^5 = 32$, but one will get 30 or 31 depending on the placement of the vertices (31 if the vertices are not equidistant around the circle, 30 if they are).

Solution 2²: For $n = 40$ the conjecture $f(n)$ is prime fails since $f(40) = 41^2$

Solution 3³: For $n = 12,055,735,790,331,359,447,442,538,767$ the conjecture $f(n)$ is not a perfect square fails since $f(12,055,735,790,331,359,447,442,538,767) = 10^{28}$

¹ I cannot remember where I found this problem.

² Schumaker, C. *Chapter Zero*, (Reading, MA: Addison-Wesley, 1997), page 66.

³ Schumaker, C. *Chapter Zero*, (Reading, MA: Addison-Wesley, 1997), page 66.