

Problems

Evaluate the integrals.

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| 1. $\int \frac{dx}{x^2(x^2+1)}.$ | 2. $\int \frac{dx}{(x+1)(x^2+4)}.$ |
| 3. $\int \frac{(3x+3) dx}{x^3-1}.$ | 4. $\int \frac{x^2 dx}{x^4-1}.$ |
| 5. $\int \frac{3 dx}{x^3+x^2+x+1}.$ | 6. $\int \frac{(x-1) dx}{x^4+x^2}.$ |
| 7. $\int \frac{dx}{(x^2+4)(x+2)^2}.$ | 8. $\int \frac{(x^2+1) dx}{x^3+x^2+x}.$ |
| 9. $\int \frac{x dx}{(x+1)(x^2+1)}.$ | 10. $\int \frac{x^3+8}{x(x^2+4)} dx.$ |
| 11. $\int \frac{(x^2-2x+2) dx}{(x^2+1)^2}.$ | 12. $\int \frac{x^3 dx}{(x^2+4)^3}.$ |
| 13. $\int \frac{dx}{x(x^2+4)^2}.$ | 14. $\int \frac{(x^2+9x+9) dx}{x^2(x^2+9)^2}.$ |
| 15. $\int \frac{x^4+4x^2+16}{(x^3-8)^2} dx.$ | 16. $\int \frac{(x^2+2x+2) dx}{x^2(x^2+2)^2}.$ |
| 17. $\int \frac{(x^2+3x+4) dx}{(x^2+1)(x^2+4)^2}.$ | 18. $\int \frac{x dx}{(x^2-1)(x^2+4)^2}.$ |
| 19. $\int \frac{dx}{x(x^2+2x+2)}.$ | 20. $\int \frac{dx}{(x^4-1)^2}.$ |

15.10*Miscellaneous Substitutions*

If none of the previous methods works, there are still a few substitutions that may be tried. One of these is the rationalizing substitution. We have already seen this type of substitution, on simple problems, in Section 15.2. Let us consider some other examples of it.

Example 1

Evaluate $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}.$

We can eliminate the square root by the substitution $u = \sqrt{x}$, or $x = u^2$, but then we still have the cube root to deal with. Similarly, we can eliminate the cube root by the substitution $u = \sqrt[3]{x}$ or $x = u^3$, but then we still have the square root to deal with. Instead of substituting $x = u^2$, so we can find its square