17. $\int \frac{(x+2)\,dx}{x^4+2x^3-3x^2}$.

18. $\int \frac{dx}{(x-1)^2(x-2)^3}$.

19. $\int \frac{dx}{x(x-1)^2(x+1)^2}$.

20. $\int \frac{dx}{(2x+1)^2(x-2)^2}$.

21. $\int \frac{\cos x\,dx}{\sin^3 x + \sin^2 x}$.

(*Hint:* let $u = \sin x$).

22. $\int \frac{\cos x\,dx}{\sin^2 x - 2\sin x - 3}$.

15.9

Partial Fractions: Quadratic Factors

Case III Nonrepeated quadratic factors:

$$\frac{P(x)}{(a_1x^2+b_1x+c_1)\cdots(a_nx^2+b_nx+c_n)} = \frac{A_1x+B_1}{a_1x^2+b_1x+c_1} + \frac{A_2x+B_2}{a_2x^2+b_2x+c_2} + \cdots + \frac{A_nx+B_n}{a_nx^2+b_nx+c_n}.$$

Example 1

Evaluate $\int \frac{dx}{x^3+x}$.

The denominator can be factored to

$$x(x^2+1),$$

but the quadratic factor cannot be factored further without using complex coefficients. Thus,

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}.$$

The constants are now evaluated exactly as in the previous section.

$$\begin{aligned} 1 &= A(x^2+1) + (Bx+C)x \\ &= (A+B)x^2 + Cx + A; \end{aligned}$$

$$A + B = 0,$$

$$C = 0,$$

$$A = 1.$$

This gives

$$A = 1, \quad B = -1, \quad \text{and} \quad C = 0.$$