MATH 302 PROBS. & STATS II WORKSHEET I 2010-01-19 NAME:__________________

(PLEASE PRINT LEGIBLY)

Use pencil only. *Limited* partial credit. **All the necessary & sufficient steps for a proof should be shown - further, justification for each step should be provided. If there is no work shown - no credit will be earned - justification must be given to claim mathematical truth!**

If an answer does not exist write **D.N.E**. (**D**oes **N**ot **E**xist) and explain why it does not exist.

0. Download and print out a copy of Math 301 Handout 1, 2, 2 ½, 3, 4, and 5.
   Math 301 Handout 5 is logically equivalent to Math 302 Handout 1.

Purchase *Mathematical Statistics* by Miller, Miller, & Freund (10^{th} edition). Prentice - Hall.
Make sure you still have *Fundamentals of Probability*, Gharamani (3^{rd} edition). Prentice - Hall.

1. The proportion of defective units produced by a manufacturing company is a random variable that is adequately approximated by beta random variable with $\alpha = 1$ and $\beta = 5$. What is the probability that the proportion of defective units is greater than 10%?

2. Suppose $X \sim \text{Ray}(x, 5)$. Find $\Pr(\mu < X \leq 2)$ where μ is of course $E[X]$.

3. Suppose the probability is $\frac{3}{4}$ that an applicant for a driver's license will pass the road test on any given attempt.

   A. The p. m. f. in this case is a : ______________________________________

   because___________________

   B. What is the probability that the applicant will pass the road test (finally) on his fourth try?

4. Prove or disprove:
Let S be a well defined sample space with E_1 , E_2 , $\wedge$ E_3 non-trivial events.
If $\Pr(E_1 \mid E_3) > \Pr(E_2 \mid E_3)$ and $\Pr(E_1 \mid E_3^C) > \Pr(E_2 \mid E_3^C)$, then $\Pr(E_1) \geq \Pr(E_2)$.

5. . Suppose $X \sim \text{Nor}(x, \mu,\sigma)$. Prove or disprove that $E[X] = \mu$

(hint: $M_X(t) = e^{(\mu t + \frac{1}{2}\sigma^2 t^2)}$)

6. Suppose $X \sim \text{Nor}(x, \mu,\sigma) \sim \text{Nor}(x, 100, 16)$.
Find $\Pr(100 \leq X)$.

7. Suppose $X \sim \text{Nor}(x, \mu,\sigma) \sim \text{Nor}(x, 100, 16)$.
Find $\Pr(116 \leq X)$.

8. Suppose $X \sim \text{Nor}(x, \mu,\sigma) \sim \text{Nor}(x, 100, 16)$.
Approximate $\Pr(100 \leq X < 140)$ to 4 decimal place accuracy.

9. Suppose $X \sim \text{Nor}(x, \mu,\sigma) \sim \text{Nor}(x, 100, 16)$.
Approximate $\Pr(X > 108)$ to 4 decimal place accuracy.

10. Suppose $X \sim \text{Wei}(x, \alpha, \beta) \ni \alpha > 0 \wedge \beta > 0$
Prove or disprove that $\mu = \alpha\, \Gamma(1 + \beta^{-1})$

11. Suppose $X \sim f(x)$ where $f(x)$ is a well defined probability mass or density function and

$M_X(t) = (1-\frac{1}{2}t)^{-5}(1-\beta t)^{-\alpha} \quad \ni t < 2$

A. Find μ B. Find σ

12. Suppose $R \sim \text{Ray}(r, 1)$
Find $\Pr(r < 1))$

13. Suppose $D \sim \text{Par}(d, \alpha) \sim \text{Par}(d, 3)$
Find $\Pr(d < \mu)$

14. Suppose $K \sim \text{Exp}(k, 2)$
Find $\Pr(k < \ln(4))$

15. Suppose $K \sim \text{Exp}(k, 2)$
Find $\Pr(k > 15 \mid k > 5)$

16. Suppose $A \sim \text{Uni}(a, -2, 5)$.
Find $\Pr(A \in (-1, 3))$

17. Suppose $A \sim \text{Uni}(a, -2, 5)$.
Find $\Pr(A \in [-1, 3))$

18. Suppose $A \sim \text{Uni}(a, -2, 5)$.
Find $\Pr(A \in \{2, 4\})$

19. Let $f((x, y, z)) = xy^3 - zx^5 + x^2yz$

A. Find $\frac{\partial^2 f}{\partial x \partial y}$ B. Find $\frac{\partial^2 f}{\partial x \partial x}$ C. Find $\frac{\partial^2 f}{\partial z \partial x}$ D. Find $\frac{\partial^3 f}{\partial z \partial x \partial y}$

20. Find $\int_1^2 \int_1^{x^2} \left(\frac{x}{y}\right) dydx$

21. Find $\int_1^2 \int_2^3 dydx$ and find $\int_2^3 \int_1^2 dxdy$

22. Find $\int_0^1 \int_{x^2}^{x} \int_{x-y}^{x+y} (x + 2y + 4z)\, dzdydx$

22. Let $\Omega = \{(x, y\}: x \in [0, 1]. \wedge\ x^2 < y \le \sqrt{x}\ \}$

Find the volume of the solid under the surface $z = x^2 + y^2$ and lying above Ω.