§7 WORKSHEET 2016 – 7
COMPUTATIONAL PROBLEMS CONDITIONAL PROBABILITY
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Theorem 6.2: 𝔹 Let S be a well defined sample space with $A \wedge B$ events where B is a non-trivial event.

Therefore it is the case that $\Pr(A) = \Pr(A \mid B) \cdot \Pr(B) + \Pr(A \mid B^c) \cdot \Pr(B^c)$. Prove the theorem.

Exercise 7.1[1]. Let S be a well-defined sample space with $A \wedge B$ events and we know that:
$\Pr(A) = 0.75$, $\Pr(B) = 0.48$; and, $\Pr(A \cup B) = 0.89$.

A. Find $\Pr(A \mid B)$ B. Find $\Pr(B \mid A)$ C. Find $\Pr(A \cap B)$ D. Find $\Pr(B \cap A)$
E. Find $\Pr(A^C \mid B)$ F. Find $\Pr(B^C \mid A)$ G. Find $\Pr(A^C \cap B^C)$ H. Find $\Pr(B^C \cap B)$

Exercise 7.2. Let S be a well-defined sample space with $A_2 \wedge B_2$ events and we know that:
$\Pr(A_2) = 0.4$, $\Pr(B_2) = 0.3$; and, $\Pr(A_2 \cap B_2) = 0.12$

A. Find $\Pr(B_2 \mid A_2)$ B. Find $\Pr(B_2^C \mid A_2^C)$ C. Find $\Pr(A_2 \cap B_2)$ D. Find $\Pr(B_2^C)$
E. Find $\Pr(A_2^C \mid B_2)$ F. Find $\Pr(B_2^C \mid A_2^C)$ G. Find $\Pr(A_2^C \cap B_2^C)$

Exercise 7.3. Let S be a well-defined sample space with $A_3 \wedge B_3$ events and we know that $\Pr(A_3) = 0.4$, $\Pr(B_3) = 0.2$; and, $\Pr(A_3 \mid B_3) = 0.2$

A. Find $\Pr(A_3 \cap B_3)$ B. Find $\Pr(B_3 \mid A_3)$ C. Find $\Pr(B_3^C)$ D. Find $\Pr(A_3 \cup B_3)$ E. Find $\Pr(B_3^C \cup B_3)$

Exercise 7.4. Suppose a pair of dice is tossed. You view the sum of the two faces up-turned.
A. Find the probability that the sum of the sides facing up is more than 5 given the sum of the sides facing up is 7 (from here on out we will term this Find the probability that the you rolled more than 5 given that you rolled a 7.
B. Find the probability that you rolled a seven given you rolled more than 5.
C. Find the probability that you rolled a less than 10 given you rolled more than 6.

Exercise 7.6. There is an urn. It contains 8 white, 3 red, 4 green, and 6 blue balls.
A. Two balls are drawn from the urn. Find the probability that both balls drawn are red.
B. Put all the balls back, shake up the urn and draw two balls (in succession) from the urn. Find the probability that the first ball is red and the second ball is green.
C. Put all the balls back, shake up the urn and draw two balls (in succession) from the urn. Find the probability that the second ball is green given the first ball is green.
D. Put all the balls back, shake up the urn and draw two balls (in succession) from the urn. Find the probability that the second ball is green given the first ball is not red.
E. Put all the balls back, shake up the urn and draw two balls (in succession) from the urn. Find the probability that one of the balls drawn is green and another is white.

Exercise 7.7. There is an urn. It contains 8 white, 3 red, 4 green, and 6 blue balls.
A. Three balls are drawn from the urn. Find the probability that all of the balls drawn are red.
B. Put all the balls back, shake up the urn and draw three balls from the urn. Find the probability that one of the balls is red and the other two are not white.
C. Put all the balls back, shake up the urn and draw three balls from the urn. Find the probability that the at least two of the balls are blue.
D. Put all the balls back, shake up the urn and draw three balls from the urn. Find the probability that the at least one of the balls is green.
E. Put all the balls back, shake up the urn and draw three balls from the urn. Find the probability that the at least one of the balls is yellow.

Last revised 2016-09-24 and initially created back around 1995.

[1] Suggestion: Perhaps use a Venn Diagramme and use the definition of the probability of W given M when $\Pr(M) \neq 0$.