§4 HANDOUT - WORKSHEET 2016 – 4A

THE GAMMA FUNCTION, PERMUTATIONS, COMBINATIONS, AND GENERALISED PRINCIPLE OF COUNTING, PART I

DR. M. P. M. M. M^C LOUGHLIN

CREATED 1995 LAST REVISED 2016

1. GAMMA

Definition 1.1. Let $x \in (0, \infty)$ **Gamma of x** is defined as the real number that is

$$\int_0^\infty t^{(x-1)} \cdot e^{-t} dt$$

Definition 1.2. The **Gamma function** is defined as

$$\Gamma : (0, \infty) \longrightarrow \mathbb{R}$$

such that

$$\Gamma(x) = \int_0^\infty t^{(x-1)} \cdot e^{-t} dt$$

Theorem 1.1. *Let* $\alpha > 1$. $\Gamma(\alpha) = (\alpha - 1) \cdot \Gamma(\alpha - 1)$.

2. FACTORAL

Definition 2.1. Let $n \in \mathbb{N}^*$ **n factoral** is defined as the integer that is

$$\int_0^\infty t^{(n)} \cdot e^{-t} dt$$

and the notation is $n!$

Definition 2.2. Let $n \in \mathbb{N}^*$. **n factoral** is defined as the natural number that is $\prod_{k=1}^{n} k$ when $n \in \mathbb{N}$;
is 1 when $n = 0$;
and, the notation is $n!$

Definition 2.3. Let $n \in \mathbb{N}$. The factoral function, f, is $f : \mathbb{N} \longrightarrow \mathbb{R}$ such that $f(n) = (n-1)!$ It is the restriction function (the sequence) $\Gamma\Big|_{\mathbb{N}}$ and the notation is $(n-1)!$

Theorem 2.1. *Let* $n \in \mathbb{N}$. $n! = n \cdot (n-1)!$.

3. Permutations

Definition 3.1. Let $n \in \mathbb{N}^*$, $k \in \mathbb{N}^*$, and $n \geq k$. The **permutations** n things ordered k at a time is defined as the integer

$$\frac{n!}{(n-k)!}$$

and the notation is ${}_nP_k \equiv P(n,k)$

4. Combinations

Definition 4.1. Let $n \in \mathbb{N}^*$, $k \in \mathbb{N}^*$, and $n \geq k$.
The **combinations** n things chosen k at a time is defined as the integer

$$\frac{n!}{k! \cdot (n-k)!}$$

and the notation is ${}_nC_k \equiv \binom{n}{k}$

Theorem 4.1. *$n \in \mathbb{N}^*$, $k \in \mathbb{N}^*$, and $n \geq k$.*

$$P(n,k) \geq \binom{n}{k}$$

Theorem 4.2. *$n \in \mathbb{N}$, $k \in \mathbb{N}$, and $n > k$.*

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Theorem 4.3. *$n \in \mathbb{N}^*$, $k \in \mathbb{N}^*$, and $n \geq k$.*

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

Theorem 4.4. *$n \in \mathbb{N}$, $k \in \mathbb{N}$, and $n \geq k$.*

$$\binom{n}{k} = \binom{n}{n-k}$$

Theorem 4.5. *$n \in \mathbb{N}$, $k \in \mathbb{N}$, and $n \geq k$.*

$$\binom{n}{0} = 1$$

Theorem 4.6. *$n \in \mathbb{N}$, $k \in \mathbb{N}$, and $n \geq k$.*

$$\binom{n}{n} = 1$$

Theorem 4.7. *$n \in \mathbb{N}$.*

$$\binom{n}{1} = n$$

Theorem 4.8. $n \in \mathbb{N}$.

$$\binom{n}{n-1} = n$$

None of these theorems are Board-worthy in MAT 301 (MAT 321 or MAT 123 maybe but not in here).

5. Computation Exercises

Compute (where such exists):

1. $\binom{7}{4}$
2. $\binom{7}{5}$
3. $\binom{8}{4}$
4. $\binom{8}{5}$
5. $\binom{4}{0}$
6. $\binom{4}{1}$
7. $\binom{4}{2}$
8. $\binom{4}{3}$
9. $\binom{4}{4}$
10. $\binom{4}{5}$
11. $\binom{-4}{-2}$
12. $\binom{14}{13}$

Last revised 2016-02-12 and initially created back around 1995