Course Objectives[1]
Probability & Statistics I
Dr. M. P. M. M. M^{c}Loughlin

Length of Course: One semester
Prerequisite: Math 224, Foundations of Mathematics, with a grade of 'C' or better; Math 283, Calculus III, with a grade of 'C' or better; or, permission of instructor.

Text: none.

Texts (supplemental): The instructor may suggest supplemental reading or exercises from a source or two (such can only be referenced with the expressed written permission of the instructor (me) and if such are used without permission the student will earn an 'F' for the semester for cheating – **that includes internet sites**):
Fundamentals of Probability, Gharamani (2^{nd} Edition). Prentice - Hall.
John E. Freund's Mathematical Statistics, Miller, Miller, & Freund (10^{th} edition). Prentice - Hall.
A First Course in Probability, Ross (6^{th} Edition). Prentice - Hall.
Schaum Outline Series: Probability, Random Variables, & Random Processes, Hsu (any ed.). McGraw - Hill.

Course Objective:

This course is designed to provide the student with an intense foundation in fundamental concepts of stochastic mathematics used in advanced mathematics. After completing the course the student should be able to work basic problems and proofs in probability, combinatorics (with special attention to those aspects of the subject most relevant to statistics), and early statistics (density, distribution, and functional random variables). The second course (Math 302) continues from whence 301 ends with a deeper discussion of mathematical statistics. It is highly recommended that a student in Actuarial Science, Engineering, Physics, Economics, Computer Science, or Applied Mathematics opt to include Math 302 in his programme of study. If the student is interested in an applied statistics course where one simply applies work others proved - this is not for you. In this course we prove as well as compute. Further, this is not a course that is computer driven, it is a theoretical course designed for an intense foundation in probability and mathematical statistics. The computer driven course at Kutztown is Math 140 (Applied Statistical Methods); the Calculus-based without proof, Biostatistics (Math 270); and, maybe the course (revived) Statistics (Math 230) which may be of interest to some students.

A student should have mastered and demonstrated the following skills after completing Math 301:

- the student is able to think logically;
- the student is able to reason and recognise patterns and be able to make conjectures;
- the student is able to use mathematical symbols;
- the student is able to use the basic ingredients of probability and elementary combinatorial analysis from a non-measure as well as measure theoretic point of view;
- the student is able to work with permutations, combinations, binomials, and multinomials;
- the student is able to prove conjectures based on the axioms of probability;
- the student is able to work with a posteriori and a priori probability;
- the student is able to explain what a proof is and discern between a valid proof and claim that a proof has been performed, but in reality has not;
- the student is able to read a proof of a statement;
- the student is able to construct a valid proof in probability theory using different methods which include: direct, proof by cases, indirect, contradiction, induction (weak and strong forms), and contraposition;
- the student is able to construct valid counterexamples to propositions which are false;

[1]This course is taught via a strong inquiry-based learning (IBL) method known as the Moore Method.

- the student is able to recognise and avoid common fallacies in arguments including begging the question, circular reasoning, affirming the conclusion, and denying the hypothesis;
- the student is able to work with discrete and continuous random variables;
- the student is able to work with probability density functions and probability distribution functions; and,
- the student is able to compute expectation, variance, and covariance.

We will be investigating a plethora of types of families of probability functions. Amongst them are the Uniform, Bernoulli, Binomial, Multinomial, Gaussian (Normal), Gaussian (Normal) bivariate, Geometric, Hypergeometric, Hazard, Chi- Squared (χ^2), Dirichlet, Erlang, Dirichlet bivariate, Error, Exponential, Gumbel, Fischer-Snedecor (F), Weibull, Cauchy, Beta, Laplace, Logistic, Lognormal, Pareto, Poisson, Rayleigh, Student (Gossett T), von Mises, Wishart, and Birnbaum families.

Outline:

Section	Title	Chapters of the Notes	Pace (class dependent)
I	Preliminaries	Chapter 1, 2	$2\frac{1}{2}$ weeks
II	Probability Theory	Chapter 2 - 3	2 weeks
III	More Probability Theory	Chapter 3 - 4	2 weeks
IV	Random Variables	Chapter 4 - 5	$3\frac{1}{2}$ weeks
V	More Random Variables	Chapter 6	$2\frac{1}{2}$ weeks
VI	Multivariate Random Variables	Chapter 7 - 8	$3\frac{1}{2}$ weeks (to end)

Content Specifics:

I. Preliminaries, simple counting, combinatorial methods, Gamma function, basic naïve probability, Axioms of Probability, sample space, events, outcomes, & claims to prove or disprove.
II. Theory of Probability, Claims about probability, and probability computation using combinatorial methods.
III. Conditional Probability, Independence, Non-independence, Bayes' Theorem, and claims about said material.
IV. Probability Mass Functions, Cumulative Distribution Functions, Moments, Specific PMFs, CDFs, and claims about PMFs or CDFs.
V Probability Density Functions, Cumulative Distribution Functions, Moments and Moment Generating Functions, Specific PDFs, CDFs, and Claims about PDFs or CDFs.
VI Jointly Distributed Random Variables, Conditional Probability revisited, Marginal Probability, Statistical Independence versus Non-independence, Joint Probability Mass or Density Functions, Cumulative Distribution Functions, Marginal Probability Mass or Density Functions, Conditional Probability Mass or Density Functions, Moments, Specific JPDFs, JCDFs, and Claims about JPDFs or JCDFs, Covariance, Correlation, and Transformations of Random variables.

Math 302 continues where Math 301 leaves off - usually somewhere with the discussion of Multivariate Random Variables. It is highly recommended that a student in Actuarial Science, Engineering, Physics, Economics, Computer Science, or Applied Mathematics opt to include Math 302 in his programme of study. Then Math 403 continues where Math 302 leaves off.