

### Worksheet 8

TUES. - THURS. CLASS RELATIONS: PARTIAL ORDERS, ETC.

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**Exercise 14.1** : Let the universe be  $U = \mathbb{R} \times \mathbb{R}$ . Let the sets  $A = \mathbb{N}_4, B = \mathbb{Z}_3^*, C = \mathbb{N}_3^*$ , and  $D = \mathbb{N}_5$ .

**14.1.1:** Let  $P \subseteq U \ni P = \{(a, b) : b \leq a\}$

Let  $R \subseteq B \times B \ni R = \{(b, c) : c > b\}$

Let  $S \subseteq C \times C \ni S = \{(c, d) : c = d \vee c = 2 \cdot d\}$

Show it is the case that  $P$  is a partial order on  $A$ ; or, provide justification why it is not the case.<sup>1</sup>

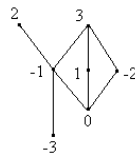
Show it is the case that  $R$  is a partial order on  $B$ ; or, provide justification why it is not the case.

Show it is the case that  $S$  is a partial order on  $C$ ; or, provide justification why it is not the case.

**14.1.2:** Let  $M \subseteq C^2 \ni P = \{(0, 0), (1, 1), (2, 2), (3, 3), (2, 0), (1, 2), (2, 3)\}$

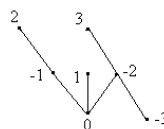
Show it is the case that  $M$  is a partial order on  $C$ ; or, provide justification why it is not the case.

**14.2.7:** Consider  $B \times B$  and the following Hasse Diagramme symbolise a partial order on  $B$ , call it  $P_1$ . Find  $P_1$  in set notation (list of the elements).



Let  $M = \{-2, -1, 1\}$ . Find the set of upper bounds,  $M_u$ , of  $M$ , the set of lower bounds,  $M_l$ , of  $M$ , the greatest lower bound and least upper bound of  $M$  under the order (if such exist).

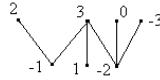
**14.2.8:** Consider  $B \times B$  and the following Hasse Diagramme symbolise partial order on  $B$ , call it  $P_2$ . Find  $P_2$  in set notation (list of the elements). Let  $M = \{-2, -1, 1\}$ . Find the set of upper



bounds,  $M_u$ , of  $M$ , the set of lower bounds,  $M_l$ , of  $M$ , the greatest lower bound and least upper bound of  $M$  under the order (if such exist).

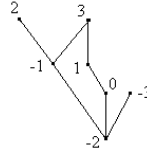
<sup>1</sup>If you have trouble with this - find some elements of  $P$  (that might help).

**14.2.9:** Consider  $B \times B$  and the following Hasse Diagramme symbolise partial order on  $B$ , call it  $P_3$ . Find  $P_3$  in set notation (list of the elements).



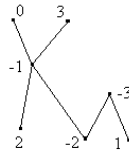
Let  $M = \{-2, -1, 1\}$ . Find the set of upper bounds,  $M_u$ , of  $M$ , the set of lower bounds,  $M_l$ , of  $M$ , the greatest lower bound and least upper bound of  $M$  under the order (if such exist).

**14.2.10:** Consider  $B \times B$  and the following Hasse Diagramme symbolise partial order on  $B$ , call it  $P_4$ . Find  $P_4$  in set notation (list of the elements).



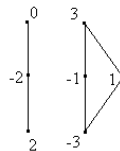
Let  $M = \{-2, -1, 1\}$ . Find the set of upper bounds,  $M_u$ , of  $M$ , the set of lower bounds,  $M_l$ , of  $M$ , the greatest lower bound and least upper bound of  $M$  under the order (if such exist).

**14.2.11:** Consider  $B \times B$  and the following Hasse Diagramme symbolise partial order on  $B$ , call it  $P_5$ . Find  $P_5$  in set notation (list of the elements).



Let  $M = \{-2, -1, 1\}$ . Find the set of upper bounds,  $M_u$ , of  $M$ , the set of lower bounds,  $M_l$ , of  $M$ , the greatest lower bound and least upper bound of  $M$  under the order (if such exist).

**14.2.12:** Consider  $B \times B$  and the following Hasse Diagramme symbolise partial order on  $B$ , call it  $P_6$ . Find  $P_6$  in set notation (list of the elements).



Let  $M = \{-2, -1, 1\}$ . Find the set of upper bounds,  $M_u$ , of  $M$ , the set of lower bounds,  $M_l$ , of  $M$ , the greatest lower bound and least upper bound of  $M$  under the order (if such exist).