

FOUNDATIONS OF MATHEMATICS  
 DR. MCLOUGHLIN'S CLASS  
 SPRING 2011  
 WORKSHEET 2  
 EXERCISE SET § 1.2

1. Learn at least *some* of the ways to translate the conditional and biconditional from handout one.

2. Memorise the laws of logic (handout 2).

3. Prove or disprove:

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|---|--|
| A. Premises: $p \rightarrow q$ , $r \rightarrow \neg q$   | Conclusion: $p \Rightarrow r$          |
| B. Premises: $p \rightarrow \neg q$ , $q$   | Conclusion: $p$                        |
| C. Premises: $p \rightarrow \neg q$ , $r \rightarrow p$ , $q$   | Conclusion: $r$                        |
| D. Premises: $p \rightarrow \neg q$ , $r \rightarrow p$ , $q$   | Conclusion: $\sim r$                   |
| E. Premises: $m \rightarrow e$ , $a \rightarrow l$ , $(e \vee l) \rightarrow \neg w$ , $w$                            | Conclusion: $\sim m \wedge \sim a$     |
| F. Premises: $w \vee (h \wedge l)$ , $w \vee h \rightarrow d$   | Conclusion: $w \vee d$                 |
| G. Premises: $p \vee q \rightarrow r$ , $s \rightarrow p \wedge a$ , $q \vee s$                                       | Conclusion: $r$                        |
| H. Premises: $p \vee q \rightarrow r \wedge s$ , $\sim r$   | Conclusion: $\sim q$                   |
| I. Premises: $p \wedge q \rightarrow r$ , $(p \rightarrow r) \rightarrow s$ , $\sim q \vee m$                         | Conclusion: $q \rightarrow s \wedge m$ |
| J. Premises: $p \wedge (q \vee s)$ , $p \rightarrow (s \rightarrow x)$ , $p \rightarrow (q \rightarrow r)$ , $\sim r$ | Conclusion: $x$                        |
| K. Premises: $a \vee (b \wedge c)$ , $b \rightarrow d$ , $c \rightarrow e$ , $d \wedge e \rightarrow f$ , $\sim a$    | Conclusion: $f$                        |
| L. Premises: $p \wedge b$ , $\sim b \vee c$   | Conclusion: $p$                        |
| M. Premises: $p \wedge b$ , $\sim b \vee c$   | Conclusion: $c$                        |
| N. Premises: $p \wedge b$ , $\sim b \vee c$   | Conclusion: $p \vee c$                 |
| O. Premises: $a \wedge b \Rightarrow c$ , $c \Rightarrow a \vee b$  | Conclusion: $\neg a$                   |
| P. Premises: $A \Rightarrow B$ , $C \Rightarrow D$ , $(B \vee D) \Rightarrow \neg E$ , $E$                            | Conclusion: $\neg(A \vee C)$           |
| Q. Premises: $A \vee B \Rightarrow D$ , $\neg B$ , $(A \wedge B) \Rightarrow C$ , $\neg C \Rightarrow \neg D$         | Conclusion: $C$                        |
| R. Premises: $B \Rightarrow A$ , $A \vee C \Rightarrow E$ , $\neg E \vee D$   | Conclusion: $B \Rightarrow D$          |
| S. Premises $A \Rightarrow B$ , $C \Rightarrow D$ , $(B \vee D) \Rightarrow \neg E$ , $E$                             | Conclusion: $\neg C$                   |

4. Prove or disprove the proposed conclusion based on the given premises:

$$p \wedge q$$

$$\sim p \vee q$$

$$\therefore q$$

5. Prove or disprove the proposed conclusion based on the given premises.

Premises:  $A \Rightarrow B$ ,  $\neg A \vee C \Rightarrow D$ ,  $\neg B$  Conclusion:  $D$

6. Claim: It is the case that  $D$  follows from the hypotheses  $A \Rightarrow B$ ,  $\neg A \vee C \Rightarrow D$ , and  $B$   
Prove or disprove the claim.

7. Given the premises  $B \Rightarrow \bar{A}$ ,  $C \vee (D \wedge A)$ ,  $\bar{C}$  Prove or disprove that  $\bar{B}$  follows as a conclusion.

8. Prove or disprove the proposed conclusion follows from the given premises.

Premises:  $P \rightarrow (R \vee Q)$ ,  $R \rightarrow \sim Q$  Conclusion:  $P \Rightarrow R$

9. Prove or disprove the proposed conclusion based on the given premises.

Premises:  $P \rightarrow (R \vee Q)$ ,  $P \vee \sim Q$ ,  $R \vee Q$ , Conclusion:  $Q$

10. Prove or disprove the proposed conclusion based on the given premises.

Premises:  $P \wedge Q \rightarrow R$ ,  $(P \Rightarrow R) \Rightarrow S$ ,  $\sim Q \vee U$ , Conclusion:  $Q \Rightarrow S \wedge U$

11.

Claim:

$$P \wedge (Q \vee S)$$

$$P \Rightarrow (S \Rightarrow U)$$

$$\sim R$$

$$\underline{P \Rightarrow (Q \Rightarrow R)}$$

$$\therefore U$$

Prove or disprove the claim.