

# Worksheet I

## FOUNDATIONAL CONCEPTS

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Definition:  $\mathbb{R} = \{x \mid \text{where } x \text{ is a point on the line}\}$ .

Let  $U = \mathbb{R}$  for the line and let  $U = \mathbb{R} \times \mathbb{R}$  for the plane.

Definition:  $\mathbb{N}^* = \{0, 1, 2, 3, \dots, (k-1), k, \dots\}$

Definition:  $\mathbb{N} = \{1, 2, 3, \dots, (k-1), k, \dots\}$

Definition:  $\mathbb{N}_k^* = \{0, 1, 2, 3, \dots, (k-1), k\}$

Definition:  $\mathbb{N}_k = \{1, 2, 3, \dots, (k-1), k\}$

Definition: Let  $a \in \mathbb{R}$ ,  $b \in \mathbb{R}$  such that  $a < b$ .

Then a **segment** is  $(a, b)$  and an **interval** is  $[a, b]$ .

A **half-segment or half-interval** is  $(a, b]$  or  $[a, b)$ .

Definition:  $\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \dots\}$

Definition:  $\mathbb{Q} = \{x \mid x = \frac{m}{n}, m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0\}$

Definition:  $\mathbb{I} = \{x \mid x \in \mathbb{R} \wedge x \notin \mathbb{Q}\}$

## Aristotelean Logic

Definition: Let  $D$  be a well-defined domain of definition. Let  $p$  be a statement.

$\neg p$  means not  $p$ . It is the negation.

When  $p$  is true,  $\neg p$  is false. When  $p$  is false,  $\neg p$  is true. That is it (there are no other possibilities).

Alternate Notation: Let  $D$  be a well-defined domain of definition. Let  $p$  be a statement.

$\neg p$  is the same as  $\bar{p}$  which is the same as  $\sim p$ .

Definition: Let  $D$  be a well-defined domain of definition. Let  $p$  and  $q$  be statements.

$p \vee q$  means  $p$  or  $q$ . It is the inclusive 'or.' It is the disjunction. When  $p$  is true and  $q$  is true,  $p \vee q$  is true.

When  $p$  is true and  $q$  is false,  $p \vee q$  is true.

When  $p$  is false and  $q$  is true,  $p \vee q$  is true.

When  $p$  is false and  $q$  is false,  $p \vee q$  is false.

That is it (there are no other possibilities).

Definition: Let  $D$  be a well-defined domain of definition. Let  $p$  and  $q$  be statements.

$p \wedge q$  means  $p$  and  $q$ . It is the conjunction. When  $p$  is true and  $q$  is true,  $p \wedge q$  is true.

When  $p$  is true and  $q$  is false,  $p \wedge q$  is false.

When  $p$  is false and  $q$  is true,  $p \wedge q$  is false.

When  $p$  is false and  $q$  is false,  $p \wedge q$  is false.

That is it (there are no other possibilities).

Definition: Let  $D$  be a well-defined domain of definition. Let  $p$  and  $q$  be statements.

$p \implies q$  means if  $p$ , then  $q$ . It is the conditional. When  $p$  is true and  $q$  is true,  $p \implies q$  is true.

When  $p$  is true and  $q$  is false,  $p \implies q$  is false.

When  $p$  is false and  $q$  is true,  $p \implies q$  is true.

When  $p$  is false and  $q$  is false,  $p \implies q$  is true.

That is it (there are no other possibilities).

Alternate Notation: Let  $D$  be a well-defined domain of definition. Let  $p$  and  $q$  be statements.

$p \implies q$  is the same as  $p \longrightarrow q$ ,  $p \Rightarrow q$ , and  $p \rightarrow q$ .

**Definition:** Let  $D$  be a well-defined domain of definition. Let  $p$  and  $q$  be statements.  
 $p \iff q$  means  $p$  if and only if  $q$ . It is the biconditional. When  $p$  is true and  $q$  is true,  $p \iff q$  is true.  
 When  $p$  is true and  $q$  is false,  $p \iff q$  is false.  
 When  $p$  is false and  $q$  is true,  $p \iff q$  is false.  
 When  $p$  is false and  $q$  is false,  $p \iff q$  is true.  
 That is it (there are no other possibilities).

### Questions and Exercises

Let  $D$  be a well-defined domain of definition. Let  $P$ ,  $Q$ , and  $R$  be statements.  
 Part A.

1. Create a complete truth table for the statement  $P \vee Q$
2. Create a complete truth table for the statement  $P \wedge Q$
3. Create a complete truth table for the statement  $\neg P \wedge \neg Q$
4. Create a complete truth table for the statement  $\neg(P \wedge Q)$
5. Create a complete truth table for the statement  $\neg(P \vee Q)$
6. Create a complete truth table for the statement  $P \wedge \neg Q$
7. Create a complete truth table for the statement  $\neg P \wedge Q$
8. Create a complete truth table for the statement  $P \vee \neg Q$

Part B.

9. Create a complete truth table for the statement  $P \implies Q$
10. Create a complete truth table for the statement  $P \wedge Q \implies P$
11. Create a complete truth table for the statement  $\neg P \implies \neg Q$
12. Create a complete truth table for the statement  $\neg(P \wedge Q) \implies P$
13. Create a complete truth table for the statement  $P \implies \neg(P \vee Q)$
14. Create a complete truth table for the statement  $P \implies P \wedge \neg Q$
15. Create a complete truth table for the statement  $\neg P \wedge Q \implies \neg Q$
16. Create a complete truth table for the statement  $P \implies P \vee \neg Q$

Part C.

17. Create a complete truth table for the statement  $P \wedge Q \vee R$
18. Create a complete truth table for the statement  $P \vee Q \wedge R$
19. Create a complete truth table for the statement  $P \implies Q \vee R$
20. Create a complete truth table for the statement  $P \vee Q \implies R$
21. Create a complete truth table for the statement  $P \implies Q \vee \neg R$
22. Create a complete truth table for the statement  $\neg P \vee Q \implies R$
23. Create a complete truth table for the statement  $P \implies R \wedge Q \vee R$
24. Create a complete truth table for the statement  $P \vee Q \implies P \wedge R$

Part D.

25. Create a complete truth table for the statement  $P \wedge Q \iff Q \wedge P$
26. Create a complete truth table for the statement  $P \implies Q \iff \neg Q \wedge \neg P$
27. Create a complete truth table for the statement  $P \implies Q \iff \neg Q \vee \neg P$
28. Create a complete truth table for the statement  $P \implies Q \iff \neg P \vee Q$
29. Create a complete truth table for the statement  $P \implies Q \iff \neg P$
30. Create a complete truth table for the statement  $\neg(P \vee Q) \iff \neg P \wedge \neg Q$

Is there a word or phrase that is unfamiliar? Then look it up!

Do not return to the next class with statements such as:

"I don't know what to do."

"I tried everything."<sup>1</sup>

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<sup>1</sup>But, did and have nothing.