Course Objectives
FOUNDATIONS OF MATHEMATICS
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Length of Course: One semester
Prerequisite: **Math 171** (Calculus I with a grade of 'C' or better) or permission of instructor. This course is best taken in the Sophomore year.

Text (required):
Introduction to Advanced Mathematics, Barnier & Feldman (3^{rd} Ed.). Prentice - Hall, 2003.

Handouts, Worksheets, Open Questions, etc. (required to be downloaded each class):
http://faculty.kutztown.edu/mcloughl/Math224.asp

Texts (optional & supplemental):
Principles of Mathematics, (web-book (in progress)), McLoughlin.
Schaum's Outline Series: Set Theory, Lipschutz, McGraw-Hill, 2000.

Course Objective:

This course is designed to provide the student with an intense foundation in fundamental concepts of mathematics used in advanced mathematics. After completing the course the student should be able to work basic problems (proofs, construction of examples, counter-examples, or argue that a claim is false) in logic, predicate calculus, set theory, with relations, functions, equivalence relations, partial orders, cardinality, and ordinality.

Furthermore, set theory is the *core* of the course with logic and predicate calculus as antecedents to the set theory, and the theory of relations and functions, number theory, cardinal and ordinal theory, or beginning topology of the $\mathbb{R}$ as consequents of set theory.

Course Objective:

It is important to note that a mathematics student needs to learn to conjecture and prove or disprove said conjecture. Ergo, learning requires doing; only through inquiry is learning achieved; and this course is meant to teach you how to do, critique, or analyse proofs, counterexamples, examples, or counter-arguments. For you to learn, you must be *active* in learning. Thus, the student must learn to understand a problem and solve it precisely, accurately, and correctly (not just 'get' an answer by 'any means'). You must learn to conjecture and prove or disprove said conjecture. One cannot learn to conjecture from a book, we learn to conjecture by conjecturing![1] One does not learn to prove claims by reading other people's proofs in a book or on the board or isprove claims by reading someone else's counterexample, we learn to prove or disprove claims by hard work, trying again and again until we succeed![2] There is no 'shortcut' to learning mathematics that is not inherently flawed or carries with it the potential for non-authentic learning (sophism). This class is a Socratic not Sophistic class.

A particular point must be made that in this course not all questions posed are answered. Many of the questions posed in the courses are left for the student to ponder during the student's matriculation and are answered (hopefully by you, the student) at a later date. Examples of proofs, counterexamples, etc. are given but *most of the actual work is done by the students.*

So, this course is designed to get you, the student, to opine, to do, and to think. You will be required to do more work than in any other course and much of it at home and by yourself with the three most important mathematical tools anyone can have: your brain, a pencil, and paper.

[1] This statement is not meant to be sarcastic but to demonstrate that there is idempotency within the meaning of the words.
[2] Ibid.

A student should have mastered and demonstrated the following skills, objectives, or proficiencies after completing Math 224 (else, the student may need to repeat the course):

- the student is able to think logically;
- the student is able to reason, recognise patterns, and make conjectures;
- the student is able to use mathematical symbols;
- the student is able to discern truth values of statements or arguments;
- the student is able to define inductive and deductive reasoning and be able to contrast the two;
- the student is able to work with existence, quantification, and other conditions;
- the student is able to understand mathematical induction and prove propositions using induction;
- the student is able to explain what a proof **is** and discern between a valid proof and claim that a proof has been performed, but in reality has not;
- the student is able to read a proof of a statement;
- the student is able to construct a valid proof using different methods which include: direct, proof by cases, indirect, contradiction, induction (weak and strong forms), and contraposition;
- the student is able to construct valid counterexamples to propositions which are false;
- the student is able to recognise and avoid common fallacies in arguments including begging the question, circular reasoning, affirming the conclusion, and denying the hypothesis;
- the student knows the basic rules of propositional logic;
- the student is able to use the basic rules of propositional logic in order to construct proofs or counter-examples;
- the student is able to perform set - theoretic operations;
- the student knows the notation and terminology of set-theory;
- the student is able to prove or disprove claims about subsets of or elements in $\mathbb{Q}$, $\mathbb{Z}$, $\vee \mathbb{N}$;
- the student is able to use Venn (Euler) diagrammes to assist in the construction of a proof or counterexample of a claim in set-theory;
- the student is able to define a binary relation between sets and to construct proofs or counter-examples about said;
- the student is able to define an equivalence relation and to construct proofs or counter-examples about said;
- the student is able to define a partial order, total order, or linear order on a set and to construct proofs or counter-examples about said;
- the student is able to define a function between sets and to construct proofs or counter-examples about said;
- the student is able to find a domain, codomain, range, corange of a function;
- the student is able to find the image and inverse image of subsets of the domain and codomain, respectively, and to construct proofs or counter-examples about said;
- the student is able to find the union and composition of functions and to construct proofs or counter-examples about said;
- the student is able to define injective, surjective, or bijective functions and to construct proofs or counter-examples about said;
- the student is able to prove statements combining the concepts of the image and inverse image of subsets of the domain and codomain, the union and composition of, or injective, surjective, or bijective functions;
- the student is able to understand cardinality of sets: denumerability, countability, infinite, finite, uncountable, and be able to give examples or counterexamples of a claim that a set is one or more of the previous;
- the student is able to define equipotent sets and prove or disprove the sets are equipotent;
- the student has a basic understanding of cardinal numbers and ordinal numbers;
- the student is able to understand Cantor's Theorem; and,
- the student is able to construct proofs in a domain of the naturals, integers, or reals.

Outline of the Course (with suggested pace):

Chapter	Title	Sections	Pace
I	Fundamentals of Logic.	Chapter 1 & handouts	1 week
II	Methods of Proof	Chapter 2 & handouts	2 weeks
III	Set Theory	Chapter 3 & handouts	3 weeks
IV	Cartesian Products, Relations, and Orders	§ 4.1, Chap. 5 &, handouts	1.5 weeks
V	Functions & Image and Inverse Image Sets	Chapter 4 & handouts	2.5 weeks
VI	Cardinality	Chapter 6 & handouts	1.5 weeks
VII	*Ordinality*	*Handouts*	*1.5 weeks*
VIII	*Number Theory of Integers*	*Chapter 8 & handouts*	*if time*
IX	*The Theory of* $\mathbb{R}$	*Chapter 9 & handouts*	*if time*

Warning$_1$: The pace is swift and one needs to keep up with the homework and material. This course is (perhaps) the toughest course in the curriculum; so do not fall behind and use every available, ethical, and practical method you have toward learning the material.

The instructor may suggest additional material from a number of sources:

Basic:

Mathematical Proofs: A Transition to Advanced Mathematics, Chartrand,

Polimeni, & Zhang, Addison-Wesley, 2003.

Sets, Functions, and Logic, Devlin, (3^{rd} Ed.) Chapman & Hall, 2004.

An Introduction to Modern Mathematics, Fine, Rand McNally, 1962.

Essentials of Mathematics, Hale, MAA, 2003.

Introductory Concepts for Abstract Mathematics, Hummel, Chapman & Hall, 2000.

Chapter Zero, Schumacher, Addison-Wesley, 1996.

A Transition to Advanced Mathematics, Smith, Eggen, & St. Andre,(4^{th} Ed.) Brooks-Cole, 1997.

How to Read and Do Proofs, Solow, (3^{rd} Ed.) Wiley, 2002.

Beyond Basic:

Classic Set Theory, Goldrei, Chapman & Hall, 1996.

Naïve Set Theory, , Halmos, D. Van Nostrand, 1960.

Elements of Set Theory, Enderton, Academic Press, 1977.

Axiomatic Set Theory, Suppes, Dover, 1972.

Set Theory: An Introduction, Vaught, (2^{nd} Ed.) Birkhäuser, 1995.

How to Prove It: A Structured Approach, Velleman, Cambridge University Press, 1994.

Warning$_2$: It needs to be said that you can read all the books till you are blue in the face, watch all the mathematicians you know do proofs for you, and you will still not have learnt the material in the course. The *only way to learn is by doing.*