BASIC CLAIMS AND EXERCISES ABOUT RELATIONS OR FUNCTIONS ACCOMPANYING HANDOUT 12 – 2010 - 4

For sets let $U = \mathbb{R}$ and for product sets let $V = \mathbb{R}\times\mathbb{R}$.

Exercise 12 – 4 – 1: Let $U = \mathbb{R}$. Let A = $\mathbb{N}_5^*$. Define the relation, R $\subseteq$ A × A, defined by

R = {(x,y) | x > y }.

A. Verify that R is a relation on A.
B. Show that R is a partial order on A (or not).
C. Show that R is an equivalence relation order on A (or not).
D. Show that R is a function from A to A (or not).

Exercise 12 – 4 – 2: Let $U = \mathbb{R}$. Let A = $\mathbb{N}_5^*$. Define the relation, $f \subseteq$ A × A, defined by

f = {(x,y) | x < y or x = y when y = 5}.

A. Verify that f is a relation on A.
B. Show that f is a partial order on A (or not).
C. Show that f is an equivalence relation order on A (or not).
D. Show that f is a function from A to A (or not).

Exercise 12 – 4 – 3: Let $U = \mathbb{R}$. Let A = $\mathbb{N}_5^*$. Define g (x) = $\sqrt{x}$

A. Determine if g is a relation on A.
B. Show that g is a partial order on A (or not).
C. Show that g is an equivalence relation order on A (or not).
D. Show that g is a well defined function from A to A (or not).

Exercise 12 – 4 – 4: Let $U = \mathbb{R}$. Let A = $\mathbb{N}_5^*$. Define h_1 as h_1 = {(0, 1), (2, 4), (3, 1), (4, 1), (5, 3)}

and assume $h_1 \subseteq$ A × A Prove or disprove that h_1 is a well defined function from A to A (or not).

Exercise 12 – 4 – 5: Let $U = \mathbb{R}$. Let A = $\mathbb{N}_5^*$. Define h_2 as

h_2 = {(0, 1), (1, 5), (2, 4), (3, 1), (4, 1), (5, 3)} and assume $h_2 \subseteq$ A × $\mathbb{N}$

Prove or disprove that h_2 is a well defined function from A to $\mathbb{N}$ (or not).

Exercise 12 – 4 – 6: Let $U = \mathbb{R}$. Let A = $\mathbb{N}_5^*$. Define h_3 as

$h_3 = \{(0, 1), (1, 5), (2, 4), (1, 3), (3, 1), (4, 1), (5, 3)\}$ and assume $h_3 \subseteq A \times \mathbb{N}$

Prove or disprove that h_3 is a well defined function from A to $\mathbb{N}$ (or not).

Exercise 12 – 4 – 7: Let $U = \mathbb{R}$. Define h_4 as

$h_4 = \{(0, 1), (1, 5), (2, 4), (1, 3), (3, 1), (4, 1), (5, 3)\}$ and assume $h_4 \subseteq \mathbb{N} \times \mathbb{N}$

Prove or disprove that h_4 is a well defined function from $\mathbb{N}$ to $\mathbb{N}$ (or not).

Exercise 12 – 4 – 8: Let $U = \mathbb{R}$. Let A = $\mathbb{N}_5^*$. Define h_2 as

$h_2 = \{(0, 1), (1, 5), (2, 4), (3, 1), (4, 1), (5, 3)\}$ and assume $h_2 \subseteq A \times \mathbb{N}$

and assume that h_2 is a well defined function from A to $\mathbb{N}$.

A. Find dom(h_2).
B. Find cor(h_2).
C. Find cod(h_2).
D. Find ran(h_2).

Exercise 12 – 4 – 9: Let $U = \mathbb{R}$. Let A = $\mathbb{N}$. Define k as $k(x) = \sqrt{x}\ \forall x \in \mathbb{N}$

Prove that k is a well defined function from $\mathbb{N}$ to $\mathbb{R}$.

Exercise 12 – 4 – 10: Let $U = \mathbb{R}$. Let A = $\mathbb{N}$. Define j as $j(x) = \sqrt{x}\ \forall x \in \mathbb{N}$

Prove that j is NOT a well defined function from $\mathbb{R}$ to $\mathbb{R}$.

Exercise 12 – 4 – 11: Let $U = \mathbb{R}$. Let A = $\mathbb{N}$. Define m as $m(x) = \sqrt{x}\ \forall x \in \mathbb{N}$

Prove that m is NOT a well defined function from $\mathbb{N}$ to $\mathbb{N}$.

Exercise 12 – 4 – 12: Let $U = \mathbb{R}$. Let A = $\mathbb{N}$. Define a as $a(x) = x^2\ \forall x \in \mathbb{N}$

Prove that a is a well defined function from $\mathbb{N}$ to $\mathbb{R}$.

Exercise 12 – 4 – 13: Let $U = \mathbb{R}$. Let A = $\mathbb{N}$. Define b as $b(x) = x^2\ \forall x \in \mathbb{N}$

Prove that b is NOT a well defined function from $\mathbb{R}$ to $\mathbb{R}$.

Exercise 12 – 4 – 14: Let $U = \mathbb{R}$. Let A = $\mathbb{N}$. Define c as $c(x) = x^2\ \forall x \in \mathbb{N}$

Prove that c is a well defined function from $\mathbb{N}$ to $\mathbb{N}$.