HANDOUT 4 ½
THE REAL AXIOMS
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The Field Axioms of $\mathbb{R}$

Axiom 1 (closure of addition): $\forall$ x, y $\in \mathbb{R}$, x + y $\in \mathbb{R}$ and (x = w $\wedge$ y = v) $\Rightarrow$ (x + y = w + v)

Axiom 2 (commutative of addition): $\forall$ x, y $\in \mathbb{R}$, x + y = y + x.

Axiom 3 (associative of addition): $\forall$ x, y, z $\in \mathbb{R}$, (x + y) + z = x + (y + z)

Axiom 4 (existence of identity of addition): $\exists$ a unique number 0 $\ni$ x + 0 = x $\forall$ x $\in \mathbb{R}$

Axiom 5 (existence of additive inverse): $\forall$ x $\in \mathbb{R}$ $\exists$ a unique number -x $\ni$ x + (-x) = 0

Axiom 6 (closure of multiplication): $\forall$ x, y $\in \mathbb{R}$, x $\cdot$ y $\in \mathbb{R}$ and (x = w $\wedge$ y = v) $\Rightarrow$ (x $\cdot$ y = w $\cdot$ v)

Axiom 7 (commutative of multiplication): $\forall$ x, y $\in \mathbb{R}$, x $\cdot$ y = y $\cdot$ x.

Axiom 8 (associative of multiplication): $\forall$ x, y, z $\in \mathbb{R}$, (x $\cdot$ y) $\cdot$ z = x $\cdot$ (y $\cdot$ z)

Axiom 9 (existence of identity of multiplication): $\exists$ a unique number 1 $\ni$ x $\cdot$ 1 = x $\forall$ x $\in \mathbb{R}$ (1 $\neq$ 0).

Axiom 10 (existence of multiplicative inverse): $\forall$ x $\in \mathbb{R}$ $\ni$ x $\neq$ 0 $\exists$ a unique number x^{-1} $\ni$ x $\cdot$ (x^{-1}) = 1

Axiom 11 (distributive of multiplication over addition): $\forall$ x, y, z $\in \mathbb{R}$, x $\cdot$ (y + z) = (x $\cdot$ y) + (x$\cdot$ z)

The Order Axioms of $\mathbb{R}$

Axiom 12 (trichotomy): $\forall$ x, y $\in$ $\mathbb{R}$, exactly one of the following relationships exists between x and y :

$x < y$, $x = y$, $\vee$ $x > y$. $[(x < y) \text{ exor } (x = y) \text{ exor } (x > y)]$

Axiom 13 (transitive): $\forall$ x, y, z $\in$ $\mathbb{R}$, $[(x < y) \wedge (y < z)] \Rightarrow (x < z)$

Axiom 14 (preservation of order under addition): $\forall$ x, y, z $\in$ $\mathbb{R}$, $(x < y) \Rightarrow (x + z < y + z)$

Axiom 15 (preservation of order for positive multiplier): $\forall$ x, y $\in$ $\mathbb{R}$, $[(x < y) \wedge (0 < z)] \Rightarrow$

$(x \cdot z < y \cdot z)$

The Completeness Axiom of $\mathbb{R}$

Axiom 16 (completeness): Let A be a subset of $\mathbb{R}$ that is bounded above. It is the case that sup(A) exists.