MATH 224 REACTION PAPER I   TEST I SPRING 2008 DR. MCLOUGHLIN

1. (From the THP) When one wishes to prove a claim such as:

$U = \mathbb{Z}$

Claim: Let x and y be integers.
If x + y is an even integer, then either both x and y are even integers or both x and y are odd integers.

Notice the logic of the claim:
Let P be " x + y is an even integer"   Q be "both x and y are even integers"
R be "both x and y are odd integers"

The claim structurally is $(P \Rightarrow (Q \vee R))$
You cannot assume or suppose Q (that is assuming part of the conclusion which a fallacy)
You cannot assume or suppose R (that is assuming part of the conclusion which a fallacy)

A handy way to do it is by contradiction:

Assume P.
Suppose $\neg(Q \vee R)$ Get $\neg Q \wedge \neg R$
Get a contradiction from having $P \wedge \neg Q \wedge \neg R$

2. (From the ICP) Consider $(\neg A \wedge A) \Rightarrow B$ it is *obviously* true since the premise is false.

3. (From the ICP) Let $M = \mathbb{Z}$, $x \in M$, $x < 6$. $5 \le x$. Therefore $A = \{x \mid x < 6 \wedge 5 \le x\}$ is:

A. [5, 6] B. (5, 6] C. [5, 6) D. (5, 6) E. {5, 6} F. None of these

Look closely at M!

4. (From the ICP) Let $U = \mathbb{R}$, $x \in U$, $y \in U$. $x \le y$. $y \le z$. Therefore, $x < z$.

This argument is:

A. a tautology B. a fallacy C. a contradiction D. invalid D. valid E. true
F. false G. sometimes true, sometimes false H. None of these

Look closely at the signs!

5. (From the ICP) When one wishes to prove a claim such as:

$U = \mathbb{N}$

Claim: If x is an odd natural number and y is an even natural number,
then x + y is an odd natural number.
Is much easier than the claim 1 above, eh?

Notice the logic of the claim:
Let P be " x + y is an odd natural number"   Q be "x is an odd natural number"
R be "y is an even natural number "

The claim structurally is $((Q \wedge R) \Rightarrow P)$

You can assume $Q \wedge R$ (that is assuming the hypothesis of the conclusion which is OK)
You can deduce R
You can deduce Q
You cannot assume or suppose P (that is assuming the conclusion which a fallacy)

6. (From the ICP) <u>Claim</u>: Let x and y be real numbers where $x < y$. Thus, $-y < -x$.
7. (From the THP) <u>Claim</u>: Let x and y be real numbers. Let $x < -1$, $1 < y$. It is the case that $x^2 \cdot y^2 > 1$.

There is no such thing as preservation of an inequality under multiplication of a negative. You cannot say such a thing as reverse an inequality under multiplication of a negative (in fact isn't hat essentially the claim in number 6?).
For number 6, you cannot assume or suppose $-y < -x$ (that is assuming the conclusion which a fallacy)
For number 7, you cannot assume or suppose $x^2 \cdot y^2 > 1$ (that is assuming the conclusion which a fallacy)

If trying an indirect argument:

for number 6, you cannot assume or suppose $-y > -x$ (that is not the negation of the conclusion)
For number 7, you cannot assume or suppose $x^2 \cdot y^2 < 1$ (that is not the negation of the conclusion)

You can suppose for number 6, $-y \not< -x$ which means $-y \geq -x$ which then leads to cases: $-y > -x \vee -y = -x$, then proceed from there, etc.

You can suppose for number 7, $x^2 \cdot y^2 \not> 1$ which means $x^2 \cdot y^2 \leq 1$ which then leads to cases: $x^2 \cdot y^2 < 1 \vee x^2 \cdot y^2 = 1$, then proceed from there, etc.

All in all, this stuff is a BLAST! Proving something you take for granted because someone once told you it or it is in a book is wonderful (but difficult - - - remember some of the 'easiest' claims are oft the 'hardest' to prove)! Because then it is yours and you are not dependent on someone else to tell you something, convince you of something, etc.