

MATH 224 FOUNDATIONS OF MATHEMATICS
 SPRING 2008
 DR. PADRAIG MCLOUGHLIN
 HANDOUT § 1.1 - 1.2
 IMPORTANT PRINCIPLES AND RULES TO MEMORISE

All prime statements P, Q, R, etc. are statements.

If P is a statement, then $\neg P$ is a statement.

If P and Q are statements, then $P \vee Q$ is a statement.

If P and Q are statements, then $P \wedge Q$ is a statement.

If P and Q are statements, then $P \Rightarrow Q$ is a statement.

If P and Q are statements, then $P \Leftrightarrow Q$ is a statement.

All of the rules should be learnt.

Idempotent Law (1) $P \vee P \equiv P$

Idempotent Law (2) $P \wedge P \equiv P$

Law of double negation $\neg(\neg P) \equiv P$ [same as $\neg(\neg P) \Leftrightarrow P$]

Or form of implication $P \Rightarrow Q \equiv \neg P \vee Q$
 (when changing from implication to or form
 reference or form; but when changing from or form to implication
 reference implication form)

Contrapositive form of implication $P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$

De Morgan Law (1) $\neg P \vee \neg Q \equiv \neg(P \wedge Q)$

De Morgan Law (2) $\neg P \wedge \neg Q \equiv \neg(P \vee Q)$

Direct Proof Law $P \wedge R \Rightarrow Q \equiv (P \Rightarrow (R \Rightarrow Q))$

Indirect Proof Law $P \wedge \neg Q \equiv \neg(P \Rightarrow Q)$ meaning that when
 on assumes $P \wedge \neg Q$ and from that we always derive a false statement the $(P \Rightarrow Q)$ must be true.

Law of the Excluded Middle (1) $P \wedge \neg P$ always false

Law of the Excluded Middle (2) $P \vee \neg P$ always true¹

Commutative Law of "or" (1) $P \vee Q \equiv Q \vee P$

¹ This is obvious, but let's take a closer look. Note $P \vee \neg P$ is logically equivalent to $\neg P \vee P$ which by the or form into implication form is $P \Rightarrow P$!!!! Now, if anyone (usually in the Social Sciences) says the Law of the Excluded Middle is an antiquated, outdated, or invalid law ask them, " 'If - - - , then - - - ' [fill in the blank] is a fallacy? "

Commutative Law of “and” (2)	$P \wedge Q \equiv Q \wedge P$
Associative Law of “or” (1)	$P \vee (Q \vee R) \equiv (P \vee Q) \vee R \equiv P \vee Q \vee R$
Associative Law of “and” (2)	$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R \equiv P \wedge Q \wedge R$
Distributive Law of “and over or” (1)	$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
Distributive Law of “or over and” (2)	$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
Law of Addition	$P \Rightarrow P \vee Q$
Law of Simplification	$P \wedge Q \Rightarrow P$
Modus Ponens	$[(P \Rightarrow Q) \wedge P] \Rightarrow Q$
Modus Tollens	$[(P \Rightarrow Q) \wedge \neg Q] \Rightarrow \neg P$
Disjunctive Syllogism	$[(P \vee Q) \wedge \neg Q] \Rightarrow P$
Hypothetical Syllogism (Transitivity)	$[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow [P \Rightarrow R]$
Assume the hypothesis of the conclusion	$(P \Rightarrow (R \Rightarrow Q)) \Rightarrow [(P \wedge R) \Rightarrow Q]$

FALLACIES²:

Asserting the conclusion $[(P \Rightarrow Q) \wedge Q] \Rightarrow P$
 (assuming the conclusion) (fallacy of the converse)
 It is actually the case that $[(P \Rightarrow Q) \wedge Q] \not\Rightarrow P$ necessarily!

Asserting the premise $(P \Rightarrow Q) \Rightarrow P$
 (assuming the premise must always be true)
 It is actually the case that $(P \Rightarrow Q) \not\Rightarrow P$ necessarily!

Fallacy of the inverse $[(P \Rightarrow Q) \wedge \neg P] \Rightarrow \neg Q$
 It is actually the case that $[(P \Rightarrow Q) \wedge \neg P] \not\Rightarrow \neg Q$ necessarily!

Fallacy (1) $P \vee Q \Rightarrow P$
 (they reversed the law of addition)

Fallacy (2) $P \Rightarrow P \wedge Q$
 (they reversed the law of simplification)

There are MANY more fallacies we could list; but, these are the most common. Avoid them!

First created: 8 April 1992 and last updated: 27 January 2010.

² For a more detailed discussion of fallacies see section, 1,4, 1,5 and 1.6 of *The Principles of Mathematics*.