Handout $4\frac{5}{8}$
The Lemmas We Need for Basic Analytic or Algebraic Proffs

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Let our universe be $\mathbb{R}$.
You may assume:

Lemma 1: $0 < 1$.

Lemma 2: Let $x \in \mathbb{R}$ It is the case that $x \cdot 0 = 0$.

Lemma 3: $(-1) \cdot (-1) = 1$.

Lemma 4: Let $x \in \mathbb{R}$. It is the case that $(-1) \cdot x = -x$

Lemma 5: Let $x \in \mathbb{R}$, $y \in \mathbb{R}$. It is the case that $x - y = x + (-y) = x + -y$.

Definition 1: Let $x \in \mathbb{R}$ It is the case that $x \cdot x = x^2$.

Law of Exponents: Let $x \in \mathbb{R}$, $a \in \mathbb{R}$, and $b \in \mathbb{R}$.

(1) $x^a \cdot x^b = x^{(a+b)}$

(2) $x^a \cdot b^a = (x \cdot b)^a$

(3) $x^a \div x^b = x^{(a-b)}$ when $x^b \neq 0$

(4) $(x^a)^b = x^{a \cdot b}$

FOR NUMBER THEORETIC CLAIMS
PROPERTIES OF NATURAL, INTEGERS, OR RATIONAL NUMBERS THAT YOU MAY ASSUME:

Closure of addition in $\mathbb{N}$: Let $m \in \mathbb{N}$ and $n \in \mathbb{N}$, then $(m + n) \in \mathbb{N}$.
Closure of multiplication in $\mathbb{N}$: Let $m \in \mathbb{N}$ and $n \in \mathbb{N}$, then $(m \cdot n) \in \mathbb{N}$.

Closure of addition in $\mathbb{Z}$: Let $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$, then $(a + b) \in \mathbb{Z}$.
Closure of subtraction in $\mathbb{Z}$: Let $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$, then $(a - b) \in \mathbb{Z}$.
Closure of multiplication in $\mathbb{Z}$: Let $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$, then $(m \cdot n) \in \mathbb{Z}$.

Closure of addition in $\mathbb{Q}$: Let $p \in \mathbb{Q}$ and $q \in \mathbb{Q}$, then $(p + q) \in \mathbb{Q}$.
Closure of subtraction in $\mathbb{Q}$: Let $p \in \mathbb{Q}$ and $q \in \mathbb{Q}$, then $(p - q) \in \mathbb{Q}$.
Closure of multiplication in $\mathbb{Q}$: Let $p \in \mathbb{Q}$ and $q \in \mathbb{Q}$, then $(p \cdot q) \in \mathbb{Q}$.
Closure of non-zero division in $\mathbb{Q}$: Let $p \in \mathbb{Q}$ and $q \in \mathbb{Q}$ where $q \neq 0$, then $\frac{p}{q} \in \mathbb{Q}$.

Odd or Even Natural Numbers

Definition 2: Let $m \in \mathbb{N}$. m is even if and only if it is the case that there is some natural number j (meaning $j \in \mathbb{N}$) such that $m = 2 \cdot j$.

Definition 3: Let $m \in \mathbb{N}$. m is odd if and only if it is the case that there is some natural number j (meaning $j \in \mathbb{N}$) such that $m = 2 \cdot j - 1$.

Odd or Even Natural Integers

Definition 4: Let $w \in \mathbb{Z}$. w is even if and only if it is the case that there is some integer p (meaning $p \in \mathbb{Z}$) such that $w = 2 \cdot p$.

Definition 5: Let $w \in \mathbb{Z}$. w is odd if and only if it is the case that there is some integer p (meaning $p \in \mathbb{Z}$) such that $w = 2 \cdot p + 1$.

Definition 5 version 2: Let $w \in \mathbb{Z}$. w is odd if and only if it is the case that there is some integer q (meaning $q \in \mathbb{Z}$) such that $w = 2 \cdot q - 1$.

If there are any other seemingly 'obvious' definitions, lemmas, theorems, corollaries, laws, etc. you wish to cite for an analytical or algebraic proof for class, please ask about it as soon as possible.