# HANDOUT 9 ½
# MATH 224 FOUNDATIONS OF MATHEMATICS
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# HANDOUT LEMMAS AND THEOREMS FOR
# ALGEBRA OF SETS
# CLAIMS

Lemma E: Let *U* be a well defined universe and A be a set. It is the case that $\varnothing \subseteq A$.[1]

Set Lemma 1: Let *U* be a well defined universe, A be a set, and B be a set.
$A \subseteq B \Leftrightarrow B^C \subseteq A^C$

Lemma U1: Let *U* be a well defined universe. It is the case that $U^C = \varnothing$.

Lemma U1: Let *U* be a well defined universe. It is the case that $\varnothing^C = U$.

Theorem 1: Let *U* be a well defined universe and A be a set. It is the case that
a) $A \not\subset A$ and b) $A \subseteq A$

Theorem 2 (Idempotency): Let *U* be a well defined universe and A be a set. It is the case that
a) $A \cup A = A$ and b) $A \cap A = A$

Theorem 3 (DeMorgan's): Let *U* be a well defined universe, A be a set, and B be a set.
It is the case that
a) $(A \cup B)^C = A^C \cap B^C$ and b) $(A \cap B)^C = A^C \cup B^C$

Theorem 4 (Absorption): Let *U* be a well defined universe, A be a set, and B be a set.
It is the case that
a) $A \cap (A \cup B) = A$ and b) $A \cup (A \cap B) = A$

Theorem 5 (Distributive): Let *U* be a well defined universe, A be a set, B be a set, and C be a set. It is the case that:
a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Theorem 6 (Transitive): Let *U* be a well defined universe, A be a set, B be a set, and C be a set. It is the case that given $A \subseteq B$ and $B \subseteq C$ implies $A \subseteq C$.

Theorem 7 (Pair-wise disjointness): Let *U* be a well defined universe, A be a set, and B be a set. It is the case that the following four sets are pair-wise disjoint:
$(A \cap B)$, $(A^C \cap B)$, $(A \cap B^C)$, and $(A^C \cap B^C)$

The rest: page 96, page 100 of Barnier & Feldman

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[1] Recall $\varnothing$ exists because of the axioms of set theory (axiom 2).