HANDOUT 4 ¾
MATH 224 FOUNDATIONS OF MATHEMATICS
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HANDOUT LEMMAS AND THEOREMS FOR ANALYTIC OR NUMBER THEORETIC CLAIMS

FOR ANALYTIC CLAIMS

Lemma 1: $0 < 1$. Lemma 2: $x \in \mathbb{R}$ It is the case that $x \cdot 0 = 0$. Lemma 3: $(-1) \cdot (-1) = 1$.

Lemma 4: $x \in \mathbb{R}$ $(-1) \cdot x = -x$ Lemma 5: $x \in \mathbb{R}$, $y \in \mathbb{R}$ $x - y = x + (-y)$.

FOR NUMBER THEORETIC CLAIMS:

Lemma 6 (Closure of $\mathbb{N}$ under +): $\forall$ a, b $\in \mathbb{N}$, a + b $\in \mathbb{N}$

Corollary 6: $\mathbb{N}$ is not closed under $-$

Lemma 7 (Closure of $\mathbb{N}$ under $\times$): $\forall$ a, b $\in \mathbb{N}$, a · b $\in \mathbb{N}$

Corollary 7: $\mathbb{N}$ is not closed under $\div$

Lemma 8 (Closure of $\mathbb{Z}$ under +): $\forall$ a, b $\in \mathbb{Z}$, a + b $\in \mathbb{Z}$

Lemma 9 (Closure of $\mathbb{Z}$ under $-$): $\forall$ a, b $\in \mathbb{Z}$, a $-$ b $\in \mathbb{Z}$

Lemma 10 (Closure of $\mathbb{Z}$ under $\times$): $\forall$ a, b $\in \mathbb{Z}$, a · b $\in \mathbb{Z}$

Corollary 10: $\mathbb{Z}$ is not closed under $\div$

Lemma 11 (Closure of $\mathbb{Q}$ under +): $\forall$ a, b $\in \mathbb{Q}$, a + b $\in \mathbb{Q}$

Lemma 12 (Closure of $\mathbb{Q}$ under $-$): $\forall$ a, b $\in \mathbb{Q}$, a $-$ b $\in \mathbb{Q}$

Lemma 13 (Closure of $\mathbb{Q}$ under $\times$): $\forall$ a, b $\in \mathbb{Q}$, a · b $\in \mathbb{Q}$

Corollary 13: $\mathbb{Q}$ is not closed under $\div$

Lemma 14A: $\mathbb{I}$ is not closed under + Lemma 14B: $\mathbb{I}$ is not closed under $-$

Lemma 14C: $\mathbb{I}$ is not closed under $\times$ Lemma 14D: $\mathbb{I}$ is not closed under $\div$