FOUNDATIONS OF MATHEMATICS
DR. MCLOUGHLIN'S CLASS
EXERCISE SET § 0

1. The Circle Problem

Let C be a circle with radius one centred at (0,0).

Step 1. Let A_1 and A_2 be points on the circle. Call them vertices. Connect all vertices with a chord (in this case A_1 and A_2 are connected by a chord). Consider the interior of the circle. Let region be defined as an interior part of the circle such that a set of chords separates it from other interior parts of the circle. Count the number of points (2), the number of chords, (1), and the number of regions (2).
Hypothesize as to the general number of points, chords, and regions. How confident are you with your predictions? Do you think your hypotheses are true or false? Why?

Step 2. Let A_1, A_2, and A_3 be points on the circle. Call them vertices. Connect all vertices with a chord (in this case A_1 and A_2 are connected by a chord, A_1 and A_3 are connected by a chord, and A_3 and A_2 are connected by a chord). Consider the interior of the circle. Let region be defined as an interior part of the circle such that a set of chords separates it from other interior parts of the circle. Count the number of points (3), the number of chords, (3), and the number of regions (4).
Hypothesize as to the general number of points, chords, and regions. How confident are you with your predictions? Do you think your hypotheses are true or false? Why?

Exercise: Do this for steps 3, 4, and 5. Hypothesize as to the general number of points, chords, and regions. How confident are you with your predictions? Do you think your hypotheses are true or false? Why?

2. The Prime Problem.

Consider $f(n) = n^2 + n + 5 \ \forall \ n \in \mathbb{N}$

Step 1. Consider $f(1)$. It is 7. What kind of natural number is it? Hypothesize as to the general rule for $f(n)$. How confident are you with your prediction? Do you think your hypothesis is true or false? Why?

Step 2. Consider $f(2)$. It is 11. What kind of natural number is it? Hypothesize as to the general rule for $f(n)$. How confident are you with your prediction? Do you think your hypothesis is true or false? Why?

Step 3. Consider $f(3)$. It is 17. What kind of natural number is it? Hypothesize as to the general rule for $f(n)$. How confident are you with your prediction? Do you think your hypothesis is true or false? Why?

Exercise: Do this for set 4, 5, 6, and 7. Hypothesize as to the general number of points, chords, and regions. How confident are you with your predictions? Do you think your hypotheses are true or false? Why?

3. The Other Prime Problem.

Consider $f(n) = n^2 + n + 41 \ \forall \ n \in \mathbb{N}$

Step 1. Consider $f(1)$. It is 43. What kind of natural number is it? Hypothesize as to the general rule for $f(n)$. How confident are you with your prediction? Do you think your hypothesis is true or false? Why?

Step 2. Consider $f(2)$. It is 47. What kind of natural number is it? Hypothesize as to the general rule for $f(n)$. How confident are you with your prediction? Do you think your hypothesis is true or false? Why?

Exercise: Do this for steps 3, 4, 5, , 6, 7, 8, 9, 10, and 11. Hypothesize as to the general nature of the numbers that are found. How confident are you with your predictions? Do you think your hypotheses are true or false? Why?

4. The Really Hard Polynomial Problem

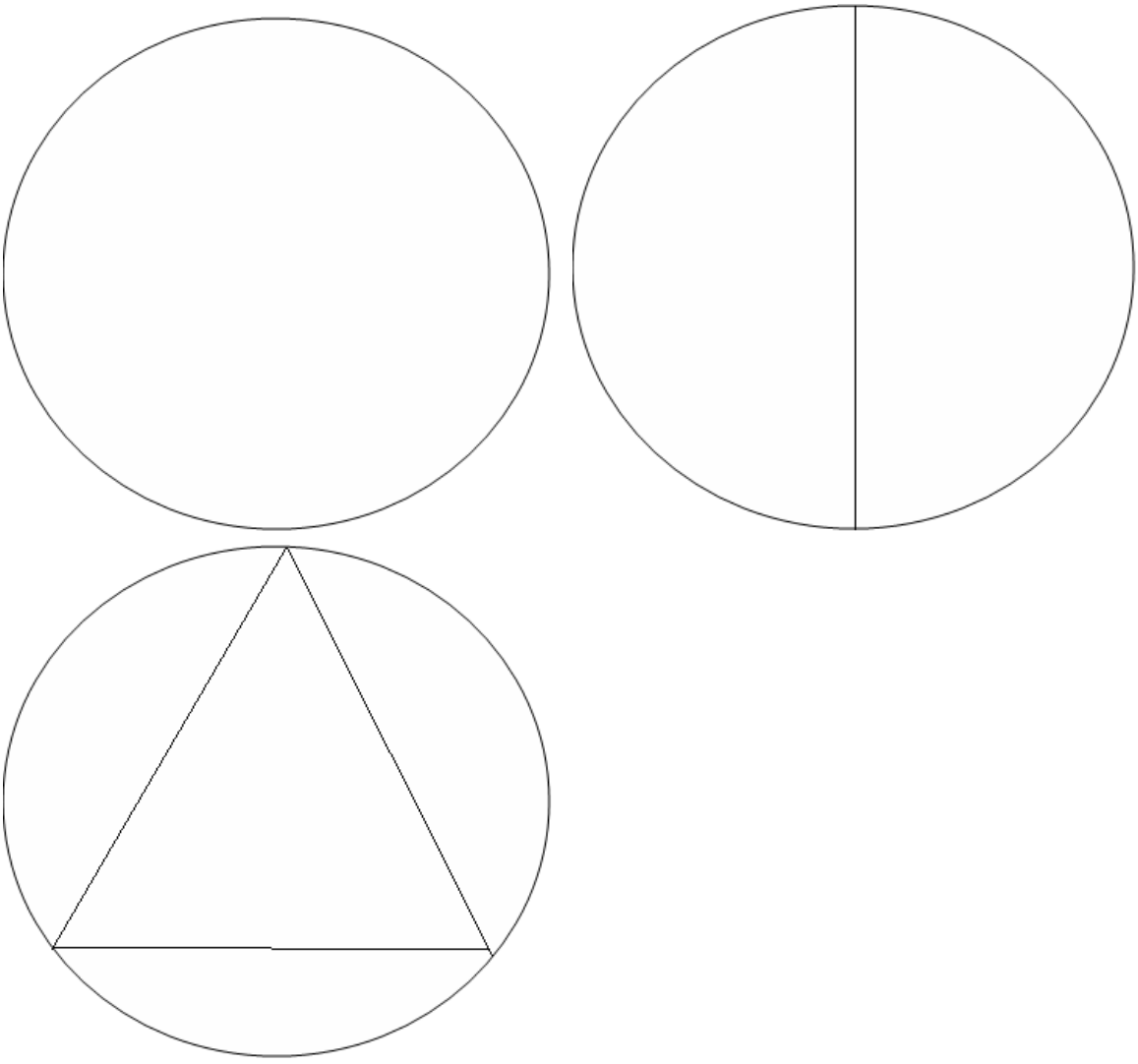
Consider $f(n) = 991n^2 + 1 \ \forall \ n \in \mathbb{N}$

Step 1. Consider $f(1)$. It is 992. What kind of natural number is it? Hypothesize as to the general rule for $f(n)$. How confident are you with your prediction? Do you think your hypothesis is true or false? Why?

Step 2. Consider $f(2)$. It is 3965. What kind of natural number is it? Hypothesize as to the general rule for $f(n)$. How confident are you with your prediction? Do you think your hypothesis is true or false? Why?

Exercise: Do this for steps 3, 4, 5, and 6. Hypothesize as to the general nature of the numbers that are found. How confident are you with your predictions? Do you think your hypotheses are true or false? Why?

Hint 1. The Circle Problem
Consider n = 6



Hint 2. The Polynomial Problem
Consider $f(40)$

Note:

$\mathbb{N}$	the natural numbers $\{1, 2, 3, 4, \ldots\}$[1]
$\mathbb{N}^*$	the non-negative integers $\{0, 1, 2, 3, 4, \ldots\}$[2]
$\mathbb{Z}$	the integers $\{0, 1, -1, 2, -2, 3, \ldots\}$
$\mathbb{Q}$	the rational numbers $\{x \mid x = \frac{m}{n}, m \in \mathbb{Z}, n \in \mathbb{Z}, \wedge n \neq 0\}$
$\mathbb{I}$	the irrational numbers
$\mathbb{R}$	the real numbers

Solution 1[3]: For n = 6 the conjecture about regions fails for $2^5 = 32$, but one will get 30 or 31 depending on the placement of the vertices (31 if the vertices are not equidistant around the circle, 30 if they are).

Solution 2[4]: For n = 5 the conjecture $f(n)$ is prime fails since $f(40) = 41^2$

Solution 2[5]: For n = 40 the conjecture $f(n)$ is prime fails since $f(40) = 41^2$

Solution 3[6]: For n = 12,055,735,790,331,359,447,442,538,767 the conjecture $f(n)$ is not a perfect square fails since $f(12{,}055{,}735{,}790{,}331{,}359{,}447{,}442{,}538{,}767) = 10^{28}$

[1] The Ordinal naturals
[2] The Cardinal naturals
[3] I cannot remember where I found this problem.
[4] Volker & Wargo. *Fundamentals of Finite Mathematics*, (Scranton, PA: Intext, 1972), page 2.
[5] Schumaker, C. *Chapter Zero*, (Reading, MA: Addison-Wesley, 1997), page 66.
[6] Schumaker, C. *Chapter Zero*, (Reading, MA: Addison-Wesley, 1997), page 66.