Handout 2A

FOUNDATIONS: LOGIC

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Important Principles and Rules Used Quite Often[1]

All prime statements P, Q, R, etc. are statements.
If P is a statement, then $\neg$ P is a statement. If P and Q are statements, then P $\vee$ Q is a statement. If P and Q are statements, then P $\wedge$ Q is a statement. If P and Q are statements, then P $\Rightarrow$ Q is a statement. If P and Q are statements, then P $\Leftrightarrow$ Q is a statement.

Law of Double Negation $\neg$ ($\neg$ P) $\equiv$ P [same as $\neg$ ($\neg$ P) $\Leftrightarrow$ P]

It is used oft and many times a person fails to realise its use.
It is also INCORRECTLY referenced in proofs by simply negating –not double negating– a statement. Pay attention to that which you wrote and that which you intended to write.

Or Form of Implication P $\Rightarrow$ Q $\equiv$ $\neg$ P $\vee$ Q

Used most oft in logic proofs. tehrefafter it is most oft used subtly to realise that in order to argue against a contional one needs the antecedent to be true and the consequent to be false.

Contrapositive Form of Implication $P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$

To say if there is a real number less than one, then there is a natural number that is 'phat' is logically equivalent to saying if there is not a natural number that is 'phat,' then there is not a real number less than one.
Remember the truth value of the two compund statements are the same – not that they must be true (for F iff F is true also).

De Morgan Law (1) $\neg$ P $\vee$ $\neg$Q $\equiv$ $\neg$ (P $\wedge$ Q)
(2) $\neg$ P $\wedge$ $\neg$Q $\equiv$ $\neg$ (P $\vee$ Q)
They are used.

Indirect Proof Law Having shown that $P \wedge \neg Q$ forces always false is logically equivalent to forcing $P \Rightarrow Q$ to have to be true.

I use this more than any other method and I opine it is crucial to good math. Others, who will be left nameless but you all know of whom I speak, disagree with me. I am right and they are wrong since this is my handout.[2]

[1]According to Dr. M.
[2]This is a fallacious form of reasoning and is used here to be funny. That I must explain such probably means it is not funny. Ç´est la vié.

Law of the Excluded Middle
(1) $P \wedge \neg P$ is always false
(2) $P \vee \neg P$ is always true[3]
A crucial law the heart of the scientific method, deductive reasoning, computer science, philosophy, and what we have today technologically, in my opinion. **Law of Addition P $\Rightarrow$ P $\vee$ Q**
An important law mostly misapplied and confused with law of simplification - make sure you realise one may add an 'or' only.

Law of Simplification P $\wedge$ Q $\Rightarrow$ P
See law of addition.

These Four Laws are used *so* frequently they should become 'second nature' and used almost without conscience thought. :

Modus Ponens [(P $\rightarrow$ Q) $\wedge$ P] $\Rightarrow$ Q

Disjunctive Syllogism [(P $\vee$ Q) $\wedge$ $\neg$Q] $\Rightarrow$ P

Hypothetical Syllogism (also called Transitivity) [(P $\rightarrow$ Q) $\wedge$ (Q $\rightarrow$ R)] $\Rightarrow$ [P $\rightarrow$ R]

Assume the Hypothesis of an Impication that is the Conclusion of an Argument

$(P \Rightarrow (R \rightarrow Q)) \Rightarrow (P \wedge R) \Rightarrow Q$

These are used so much and in long strings of arguments. It is vital one understands each one and each one's applications and correct usage.

Not as much but still worth noting:
Modus Tollens $[(P \rightarrow Q) \wedge \neg Q] \Rightarrow \neg P$

Another reiteration of FALLACIES to avoid because they are so easy a thing to do and so easy to fall into for they 'fix' problems people have in proofs (make proofs seem to work out).

Asserting the conclusion (assuming the conclusion) (fallacy of the converse)
Asserting a premise of an implication
Fallacy of the inverse
Fallacy (1) P $\vee$ Q $\Rightarrow$ P (they reversed the law of addition)
Fallacy (2) P $\Rightarrow$ P $\wedge$ Q (they reversed the law of simplification).

[3] This is obvious, but let's take a closer look. Note $P \vee \neg P$ is logically equivalent to $\neg P \vee P$ which by the or form into implication form is $P \Rightarrow P$!!!! Now, if anyone (usually in the Social Sciences) says the Law of the Excluded Middle is an antiquated, outdated, or invalid law ask them, " 'If - - - -, then - - - -' [fill in the blank] is a fallacy?"