Worksheet 9

FUN WITH FUNCTIONS

DR. M. P. M. M. MCLOUGHLIN

SPRING 2011

We started this section of the course building from sets, power sets, collections, to the more general product sets; then to relations, equivalence relations, and partial orders. Now we are at the crecendo, the pinnacle, the best stuff on Earth (sorry, Snapple).
Recall: Let U and V be well defined universes. Let $W = U \times V$ be the well defined universe for product sets. Assume $U = \mathbb{R}$ and $V = \mathbb{R} \Longrightarrow W = \mathbb{R}^2$.

Claim 9.1 : Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ be defined by $f(n) = 2 \cdot n + 1$.
Assume $f \subseteq \mathbb{N} \times \mathbb{N}$ (which implies f is a well defined relation from $\mathbb{N}$ to $\mathbb{N}$).
It is the case that f is a well defined function from $\mathbb{N}$ to $\mathbb{N}$.
Prove or disprove the claim.

Claim 9.2 : Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ be defined by $f(n) = 2 \cdot n + 1$.
It is the case that f is a well defined injection from $\mathbb{N}$ to $\mathbb{N}$.
Prove or disprove the claim.

Claim 9.3 : Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ be defined by $f(n) = 2 \cdot n + 1$.
It is the case that f is a well defined surjection from $\mathbb{N}$ to $\mathbb{N}$.
Prove or disprove the claim.

Claim 9.4 : Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ be defined by $f(n) = 2 \cdot n + 1$.
It is the case that f is a well defined bijection from $\mathbb{N}$ to $\mathbb{N}$.
Prove or disprove the claim.

Claim 9.5 : Let $g : \mathbb{Z} \longrightarrow \mathbb{R}$ be defined by $g(z) = \frac{2}{z}$.
It is the case that g is a well defined function from $\mathbb{Z}$ to $\mathbb{R}$.
Prove or disprove the claim.

Claim 9.6 : Let $h : \mathbb{Z} \longrightarrow \mathbb{Z}$ be defined by $h(w) = 3 \cdot \mathbb{Z} + 4$.
It is the case that h is a well defined function from $\mathbb{Z}$ to $\mathbb{Z}$.
Prove or disprove the claim.

Claim 9.7 : Let $j : \mathbb{Z} \longrightarrow \mathbb{Z}$ be defined by $j(x) = 3 \cup 4 \cdot x$.
It is the case that h is a well defined function from $\mathbb{Z}$ to $\mathbb{Z}$.
Prove or disprove the claim.

Claim 9.8 : Let $k : \mathbb{N} \longrightarrow \mathbb{Z}$ be defined by $k(x) = \frac{1}{x}$.
It is the case that k is a well defined function from $\mathbb{N}$ to $\mathbb{Z}$.
Prove or disprove the claim.

Claim 9.9 : Let $m : (\mathbb{N} \times \mathbb{N}) \longrightarrow \mathbb{N}$ be defined by $m((a, b)) = 2^a \cdot 3^b$.
It is the case that m is a well defined function from $\mathbb{N}^2$ to $\mathbb{N}$.
Prove or disprove the claim.

Claim 9.10 : Let $f(p) = 2 \cdot p$.
It is the case that f is a well defined function.
Prove or disprove the claim.

Notation can be (and is oft) abused or misunderstood. Using mathematical notation that suggests something does not make it so; something is or is not - but not both. A claim is either true or false but not both (conditionally on an axiom system, logic, etc.)