MATH 224 FOUNDATIONS OF MATHEMATICS
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HANDOUT § 1.1 - 1.2
SOME OF THE VARIATIONS OF THE CONDITIONAL AND BICONDITIONAL
(WORDS AND SYMBOLS)

Conditional $P \Rightarrow Q$, $P \to Q$, $Q \Leftarrow P$, and $Q \leftarrow P$ *all* mean 'If P, then Q.'

Now, the various mathematical ways to say the same thing are as follows:

If P, then Q.
Q, if P
P hence Q
Q whence P
P is a sufficient condition for Q
Q is a necessary condition for P
P only if Q
If not Q, then not P
P implies Q
Not P, or Q
etc.

Biconditional $P \Leftrightarrow Q$, $P \leftrightarrow Q$, and $P \equiv Q$ *all* mean 'If P, then Q; and, if Q then P.'

Now, the various mathematical ways to say the same thing are as follows:

If P, then Q; and, if Q then P.
P if and only if Q
P iff Q
P is necessary and sufficient for Q
P and Q are logically equivalent
If P then Q and if Q then P.
etc.

Also the order of operation for the symbols is:

Highest precedence parentheses ( )
not ~, ¬, or $\overline{}$
and - or (from left to right) $\wedge$ $\vee$
conditional $\Rightarrow$ $\to$
biconditional $\Leftrightarrow$ $\leftrightarrow$
Lowest precedence

Note when two symbols of equal precedence are connecting, then precedence is from left to right (e.g.: $P \Rightarrow Q \Rightarrow R$ means $(P \Rightarrow Q) \Rightarrow R$);
but non-equal precedence *does not follow* left to right but by order of precedence (e.g.: $P \Rightarrow Q \wedge R$ means $P \Rightarrow (Q \wedge R)$ and $P \wedge Q \Rightarrow R$ means $(P \wedge Q) \Rightarrow R$).

A Truth Table:

$[(A \wedge B \rightarrow D) \wedge D \rightarrow C \wedge A] \Rightarrow [\neg A \vee \neg B]$.

1. Note there are 4 prime statements so the number of rows of the truth table required is $2^4 = 16$ ($2^{(\text{number of prime statements})}$).
2. Note there 9 connectives so the number of columns required for the truth table is $9 + 4 = 13$ (connectives plus number of prime statements).

Note by order of operations, one can add in parentheses to more clearly see the statement that one is trying to test for validity.

$[(A \wedge B \rightarrow D) \wedge D \rightarrow C \wedge A] \Rightarrow [\neg A \vee \neg B] \equiv$
$[\{(\{A \wedge B\} \rightarrow D) \wedge D\} \rightarrow C \wedge A] \Rightarrow [\neg A \vee \neg B] \equiv$
$[\{(\{A \wedge B\} \rightarrow D) \wedge D\} \rightarrow (C \wedge A)] \Rightarrow [\neg A \vee \neg B]$

Note by order of operations, you can still choose to do different things first since there are multiple parentheses; but, let us do the parentheses left to right.

The first 4 columns are the prime statements in order of use rather than in alphabetical order.
Column 5 will have the truth values for $A \wedge B$
based on the previous columns [1 and 2 only].
Column 6 will have the truth values for $(\{A \wedge B\} \rightarrow D)$
based on the previous columns [5 and 3].
Column 7 will have the truth values for $\{(\{A \wedge B\} \rightarrow D) \wedge D\}$
based on the previous columns [6 and 3].
Column 8 will have the truth values for $(C \wedge A)$
based on the previous columns [4 and 1].
Column 9 will have the truth values for $[\{(\{A \wedge B\} \rightarrow D) \wedge D\} \rightarrow (C \wedge A)]$
based on the previous columns [7 and 8].
Column 10 will have the truth values for $\neg A$
based on the previous column [1].
Column 11 will have the truth values for $\neg B$
based on the previous column [2].
Column 12 will have the truth values for $[\neg A \vee \neg B]$
based on the previous columns [10 and 11].
Column 13 will have the truth values for $[\{(\{A \wedge B\} \rightarrow D) \wedge D\} \rightarrow (C \wedge A)] \Rightarrow [\neg A \vee \neg B]$
based on the previous columns [9 and 12].

Column 13 gives the truth values for the statement.

Statements:

Definition 1.1: A statement is a **tautology** (tautological) if for every truth-value of the prime statements, the result is true.

Definition 1.2: A statement is a **fallacy** (fallacious) if there is at least one condition of the prime statements for which the statement is true ***and*** there is at least one condition of the prime statements for which the statement is false.

Definition 1.3: A statement is a **contradiction** (contradictory) if for every truth-value of the prime statements, the result is false.

Arguments:

Definition 1.4: An argument is **valid** (the argument is tautological) if it is a tautology.

Definition 1.5: An argument is **invalid** (the argument may be fallacious or contradictory) if it is a fallacy or a contradiction.

Note for Understanding: We wish to create arguments that are **valid**; so, we which to argue tautologically through reason. One can argue with tautological statements or contradictions; but *never ever* with fallacies. Fallacies lead nowhere.