

MATH 224 FOUNDATIONS  
HANDOUT VIII  
DR. MCLOUGHLIN'S CLASS  
THE AXIOMS OF SET THEORY

**Axiom 1) The Axiom of Extension** Two sets are equal *iff* they have the same elements.

**Axiom 2) The Axiom of Null** There exists a set with no elements, call it  $\emptyset$ .

**Axiom 3) The Axiom of Pairing** Given any sets A and B, there exists a set C whose elements are A and B.

**Axiom 4) The Axiom of Union** Given any set A, the union of all elements in A is a set.

**Axiom 5) The Axiom of Power Set** Given any set A, there exists a set B consisting of all the subsets of A.

**Axiom 6) The Axiom of Separation** Given any set A and a sentence  $p(a)$  that is a statement for all  $a \in A$ , then there exists a set  $B = \{ a \in A : p(a) \text{ is true} \}$ .

**Axiom 7) The Axiom of Replacement** Given any set A and a function  $f$  defined on A, the image  $f(A)$  is a set.

**Axiom 8) The Axiom of Infinity** There exists a set A such that  $\emptyset \in A$ , and whenever  $a \in A$ , it follows that  $a \cup \{a\} \in A$ .

**Axiom 9) The Axiom of Regularity** Given any non-empty set A, there exists an  $a \in A$  such that  $a \cap A = \emptyset$ .

**Axiom 10) The Axiom of Choice** Given any non-empty set A whose members are pairwise disjoint non-empty sets, there exists a set B consisting of exactly one element taken from each set belonging to A.