MATH 224
FOUNDATIONS OF MATHEMATICS
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HANDOUT 9
§ 3.3 ALGEBRA OF SETS & LAWS TO MEMORISE AFTER YOU HAVE PROVEN THEM

Let U designate a well defined universe and A, B, and C sets within the universe. We shall denote the complement of the set A as A^C or as A'

Law of the double complement $(A^C)^C = A$

Note: recall from logic: $(\neg(\neg P) \equiv P)$. The student should verify these laws have corresponding laws of logic.

Lemma E: $\forall$ set $A \subseteq U$, $(\emptyset \subseteq A)$

Contrapositive form of subset $A \subseteq B \equiv B' \subseteq A' \equiv A' \supseteq B'$

De Morgan Law (1) $A' \cap B' \equiv (A \cup B)'$
De Morgan Law (2) $A' \cup B' \Leftrightarrow (A \cap B)'$

Law of the Excluded Middle (1) $x \in A \cap A^C \equiv$ always false
Law of the Excluded Middle (2) $x \in A \cup A^C \equiv$ always true
Law of the Excluded Middle (3) $A \cap A^C = \emptyset$
Law of the Excluded Middle (4) $A \cup A^C = U$

Commutative Law of "or" (1) $A \cup B = B \cup A$
Commutative Law of "and" (2) $A \cap B = B \cap A$

Associative Law of "or" (1) $A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C$
Associative Law of "and" (2) $(A \cap B) \cap C = (A \cap B) \cap C = (A \cap B) \cap C$

Distributive Law of "and over or" (1) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Distributive Law of "or over and" (2) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Idempotent Law (1) $A \cup A = A$
Idempotent Law (2) $A \cap A = A$

Identity Law (1) $A \cup U = U$
Identity Law (2) $A \cap \emptyset = \emptyset$
Identity Law (3) $A \cap U = A$
Identity Law (4) $A \cup \emptyset = A$

Complement Law (1) $U^C = \varnothing$
Complement Law (2) $\varnothing^C = U$

The Illustration of the Laws of Logic Applied to Set Theory:

Law of Addition $x \in A \Rightarrow x \in (A \cup B)$

Law of Simplification $x \in A \cap B \Rightarrow x \in A$

Modus Ponens $x \in A \wedge A \subseteq B \Rightarrow x \in B$

Modus Tollens $x \notin B \wedge A \subseteq B \Rightarrow x \notin A$

Disjunctive Syllogism $(x \in (A \cup B)) \wedge (x \notin B) \Rightarrow x \in A$

Hypothetical Syllogism $A \subseteq B \wedge B \subseteq C \Rightarrow A \subseteq C$
(Transitivity)

Statement:	It's Negation:
$\forall x \in U, (x \in A \Rightarrow x \in B)$	$\exists x \in U, (x \in A \wedge x \notin B)$
$\exists x \in U, (x \in A \Rightarrow x \in B)$	$\forall x \in U, (x \in A \wedge x \notin B)$
$A \subseteq B$	$A \nsubseteq B$ (NOT $A \not\subset B$ which is too strong)

Page 100 of Barnier and Feldman: S14 and S15 are wrong! Please correct them.