

**Worksheet 5**  
**PRODUCT SETS**  
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We started this section of the course building from sets to the more general product sets. Recall: Let  $U$  be a well defined universe. Let  $V$  be the well defined universe,  $\mathcal{P}(U)$ . Let  $U$  be a well defined universe. Let  $A$  be a set whose points,  $p$  are elements from  $U$ . Let  $B$  be a set whose points,  $b$  are elements of  $V$ . We defined the universe  $W = U \times V$ .

**Def. 12.1** : Let the universe  $W = U \times V$  be defined from the well defined universes  $U$  and  $V$  such that  $A \subseteq U$  whilst  $B \subseteq V$ . The product set  $A$  with  $B$  is the set  $A \times B = \{(a, b) : a \in A, b \in B\}$ .

So far this semester we have considered theorems about points,  $p$ , sets,  $A$ , power sets,  $\mathcal{P}(A)$ , and collections,  $\Omega$ . Now we have some claims about product sets to consider.

We can make our lives easy by letting  $U$  be a well defined universe and let  $V$  be a well defined universe. Let  $W$  be the well defined universe  $U \cup V$ .

**Claim 12.1** : Let  $M = W \times W$  Let  $A, B, C, \wedge D$  be non-empty sets.  
 It is the case that  $(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D)$ .

**Claim 12.2** : Let  $M = W \times W$  Let  $A, B, C, \wedge D$  be non-empty sets.  
 It is the case that  $(A \cap B) \times (C \cap D) \subseteq (A \times C) \cap (B \times D)$ .

**Claim 12.3** : Let  $M = W \times W$  Let  $A, B, C, \wedge D$  be non-empty sets.  
 It is the case that  $(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$ .

**Claim 12.4** : Let  $M = W \times W$  Let  $A, B, C, \wedge D$  be non-empty sets.  
 It is the case that  $(A \times B) - (C \times D) \subseteq (A - C) \times (B - D)$ .

**Claim 12.5** : Let  $M = W \times W$  Let  $A, B, C, \wedge D$  be non-empty sets.  
 It is the case that  $(A - B) \times (C - D) \subseteq (A \times C) - (B \times D)$ .

**Claim 12.6** : Let  $M = W \times W$  Let  $A, \wedge B$  be non-empty sets.  
 It is the case that  $(A \times B)^C = A^C \times B^C$ .