

**Handout**  $4\frac{63}{64}$

The LeVan and Benton Definitions, Claims, Theorems, or Lemmas Which  
May Be of Use

for Basic Analytic, Number Theoretic, or Algebraic Proofs

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Let our universe be  $\mathbb{R}$ .

$$U = \mathbb{R}$$

Definition L.1: Let  $x \in \mathbb{R}$ . It is the case that  $x > 0$  iff  $x$  is positive.

Definition L.2: Let  $x \in \mathbb{R}$ . It is the case that  $x < 0$  iff  $x$  is negative.

Definition L.3: Let  $x \in \mathbb{R}$  such that  $x \geq 0$ . We define  $\sqrt{x}$  to be the real number  $y$  where  $y \in [0, \infty)$  such that  $y^2 = x$ .

Prove the following:

Lemma L.1: Let  $x \in \mathbb{R}$ . It is the case that  $x$  is negative implies  $-x$  is positive.

Prove or disprove the following:

Claim L.1: Let  $x \in \mathbb{R}$ . It is the case that  $x^0 = 1$

Prove the following:

Theorem L.1: Let  $x \in \mathbb{R}$  and let  $y \in \mathbb{R}$ , it is the case that  $x + (-y) = x - y$ .

Lemma B.1 (Benton's Lemma): Let  $x \in \mathbb{R}$ , it is the case that  $x > 0 \implies \sqrt{x} > 0$ .

Corollary B.1: Let  $x \in \mathbb{R}$  such that  $\sqrt{x}$  exists. It is the case that  $\sqrt{x}$  is unique.

The challenge: Try to prove the lemmas, corrolaries, or theorems. Try to prove or disprove the claims.