

Worksheet 8 Integration With Partial Fraction Decomposition

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SPRING OF 2016

Recall we are using the Fundamental Theorem of Arithmetic, the Fundamental Theorem of Algebra, along with the theorem of equality of rationals with the same denominator and equality of polynomials.

The partial fraction decomposition is for a **fraction that is in reduced form**. So, for example:

$$\frac{5x^2 + 7x - 2}{x^4 + 2x^3 - 7x^2 + 4x}$$

must be factored

$$\frac{(x + 2)(x + 1)(x - 1)}{x(x - 1)^2(x + 4)}$$

which is

$$\frac{5x^2 + 7x - 2}{x^4 + 2x^3 - 7x^2 + 4x} = \frac{(x + 2)(x + 1)}{x(x - 1)(x + 4)}$$

So, the partial fraction decomposition can now be done:

$$\frac{5x^2 + 7x - 2}{x^4 + 2x^3 - 7x^2 + 4x} = \frac{a}{x} + \frac{b}{x - 1} + \frac{c}{x + 4}$$

Exercises:

Exercise 8.1. *Finish the problem from class (repeating linear factors).*

e.g.: Consider

$$\int \frac{x^3 + 2x^2 - x - 2}{x(x - 1)^2(x + 4)} dx$$

integrate it

Partial fraction decomposition:

$$\frac{5x^2 + 7x - 2}{x(x - 1)^2(x + 4)} = \frac{a}{(x - 1)} + \frac{b}{(x - 1)^2} + \frac{c}{(x)} + \frac{d}{(x + 4)}$$

$$\frac{5x^2 + 7x - 2}{x(x - 1)^2(x + 4)} = \frac{a(x - 1)x(x + 4)}{x(x - 1)^2(x + 4)} + \frac{bx(x + 4)}{x(x - 1)^2(x + 4)} + \frac{c(x - 1)^2(x + 4)}{x(x - 1)^2(x + 4)} + \frac{d(x - 1)^2(x)}{x(x - 1)^2(x + 4)}$$

$$\frac{5x^2 + 7x - 2}{x(x - 1)^2(x + 4)} = \frac{a(x - 1)x(x + 4) + bx(x + 4) + c(x - 1)^2(x + 4) + d(x - 1)^2(x)}{x(x - 1)^2(x + 4)}$$

$$\frac{0x^3 + 5x^2 + 7x - 2}{x(x - 1)^2(x + 4)} = \frac{a(x^3 + 3x^2 - 4x) + b(x^2 + 4x) + c(x^3 + 2x^2 - 7x + 4) + d(x^3 - 2x^2 + x)}{x(x - 1)^2(x + 4)}$$

$$\begin{array}{rcll} 0 & = & a & + c & + d \\ 5 & = & 3a & + b & + 2c & - 2d \\ 7 & = & -4a & + 4b & - 7c & + d \\ -2 & = & & & 4c & \end{array}$$

I think the solution is $a = 1, b = 2, c = -1/2,$ and $d = -1/2$

If so, to finish you must solve:

$$\int \frac{1}{(x-1)} dx + \int \frac{2}{(x-1)^2} dx + \int \frac{-1/2}{x} dx + \int \frac{-1/2}{(x+4)} dx$$

Exercise 8.2. Find

A. $\int \frac{2x}{(x-1)(x+5)} dx$

B. $\int \frac{3}{(x-1)(x+5)} dx$

C. $\int \frac{x^2-1}{(x-1)(x+5)} dx$

D. $\int \frac{x^2-4x-5}{(x-2)(x-4)(x-5)} dx$

E. $\int \frac{2x^2}{(x-1)^3} dx$

F. $\int \frac{4x-1}{(x-2)(x-4)(x-5)} dx$

Exercise 8.3. Find $\int \frac{2x^2}{(x-1)(x+5)} dx$

Exercise 8.4. Find $\int \frac{2x^2}{(x-1)(x+5)} dx$

Exercise 8.5. Find $\int \frac{x^2+x+1}{(x-2)(x-4)^2} dx$