Worksheet Zero

Pre-calculus and Calculus I Concepts

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Definition: $\mathbb{R} = \{x|$ where x is a point on the line$\}$.

Let $U = \mathbb{R}$ for the line and let $U = \mathbb{R} \times \mathbb{R}$ for the plane.

Definition: $\mathbb{N}^* = \{0, 1, 2, 3, \ldots, (k-1), k, \ldots\}$

Definition: $\mathbb{N} = \{1, 2, 3, \ldots, (k-1), k, \ldots\}$

Definition: $\mathbb{N}_k^* = \{0, 1, 2, 3, \ldots, (k-1), k\}$

Definition: $\mathbb{N}_k = \{1, 2, 3, \ldots, (k-1), k\}$

Definition: Let $a \in \mathbb{R}$, $b \in \mathbb{R}$ such that $a < b$.
Then a **segment** is (a, b) and an **interval** is $[a, b]$.
A **half-segment or half-interval** is $(a, b]$ or $[a, b)$.

Definition: $\mathbb{Z} = \{0, 1, -1, 2, -2, 3, , -3 \ldots\}$

Definition: $\mathbb{Q} = \{x | x = \frac{m}{n}, m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0\}$

Definition: $\mathbb{I} = \{x | x \in \mathbb{R} \wedge x \notin \mathbb{Q}\}$

Questions and Exercises

You may not use calculators, computers, etc. No help from any person other than yourself and from any notes other than your own. You may use other books: from the library, from a professor, etc. Use pencil only. All the necessary & sufficient steps for a solution should be shown - further, justification for each step should be provided. If an answer does not exist write D.N.E. (Does Not Exist) and explain why it does not exist.

Let $U = \mathbb{R} \times \mathbb{R}$

1. Graph the following systematically. Beegin with $f(x) = x^2$ where $f : \mathbb{R} \longrightarrow \mathbb{R}$ and 'track' the point $(0, 0)$ through the different stages of the systematic graphing or begin with $g(x) = x^3$ where $g : \mathbb{R} \longrightarrow \mathbb{R}$ and 'track' the point $(0, 0)$ through the different stages of the systematic graphing (see systematic graphic review handout on my Math 181 web-page for a review or reminder of this material):

A. $f_A(x) = \frac{3}{4}(x^2 - 5)$ B. $f_B(x) = -2(x + \frac{\pi}{2})^2 + 4$

C. $g_C(x) = \frac{10}{3}(x - 2)^3 - 1$ D. $g_D(x) = -\ln 2(x)^3 + \pi$

2. Graph the following systematically. Begin with $k(x) = \sqrt{x}$ where $k : [0, \infty) \longrightarrow \mathbb{R}$ and 'track' the point $(0, 0)$ through the different stages of the systematic graphing; further note the domain and codomain of the new (final) function (see systematic graphic review handout on my Math 181 web-page for a review or reminder of this material):

A. $k_A(x) = \sqrt{x - 2}$ B. $k_B(x) = -\sqrt{x} + 4$

C. $k_C(x) = \frac{1}{3}\sqrt{x + 1} + 5$ D. $k_D(x) = \sqrt{x + 2}$

3. Graph the following systematically. Begin with $j(x) = e^x$ where $j : \mathbb{R} \longrightarrow (0, \infty)$ and 'track' the point $(0, 1)$ through the different stages of the systematic graphing; further note trhe domain and **range** of the new (final) function (see systematic graphic review handout on my Math 181 web-page for a review or reminder of this material):

A. $j_A(x) = e^{(x-2)}$ B. $j_B(x) = e^x - 2$

C. $j_C(x) = e^{-x} + 5$ D. $j_D(x) = e^{x-2} + 3$

E. $j_E(x) = \ln x + 5$

4. Graph the following systematically. Begin with $p(x) = \cos x$ where $p : \mathbb{R} \longrightarrow \mathbb{R}$ and 'track' the point $(0, 1)$ through the different stages of the systematic graphing; further note the domain and **range** of the new (final[1]) graph (see systematic graphic review handout on my Math 181 web-page for a review or reminder of this material):

A. $p_A(x) = 7\cos(2x - \pi)$ B. $p_B(x) = \cos x + \frac{\pi}{6} - 2$

C. $p_C(x) = \cos(-x) + 5$ D. $p_D(x) = \cos\left(x - \frac{\pi}{2}\right) + 3$

5. Find $\dfrac{dy}{dx}$ for the following where each is a plane curve for the values of x where such exists.[2]

SIMPLIFY or USEFULFY the result in A through D

A. $y = (x-3)^8(2x+5)^4$ B. $y = \sqrt{\frac{(2x-3)^3}{(2x+3)^3}}$

C. $y = \dfrac{x^2 - 2\sqrt{x^3}}{\sqrt{x}}$ D. $x^2 + 2y^2 = \pi^2$

E. $y = \sqrt[4]{x - 3x^2}$ F. $y = \dfrac{\sin(x) + 2}{1 - \cos(x)}$

G. $y^2 = (\sec(x))(\cos(x))$ H. $y = 6x\sin(x)$

I. $y = (\ln(x))(\tan(x))$ J. $y \cdot x = 6e^x \cdot \sin(x)$

6. Find the following where each is a function $f : D \longrightarrow \mathbb{R}$ where $D \subseteq \mathbb{R}$ specified per problem.

A. Find $\dfrac{dy}{dx}$ where $f(x) = (3x-5)^2 \cdot (2x+5)^3$ and $D = \mathbb{R}$

B. Find $f'(4)$ where $f(x) = (3x-5)^2 \cdot (2x+5)^3$ and $D = \mathbb{R}$

C. Find $\dfrac{dy^2}{d^2x}$ where $f(x) = \sqrt[4]{x^3}$ and $D = [0, \infty)$

D. Find the equation of the tangent line to $f(x) = e^x \cdot x^2$ at $x = 2$ such that $D = [1, 4]$

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[1] Show at least two periods beyond the y-axis both left and right of the y-axis, please.

[2] For example, the standard graph for $y = \sqrt{x}$ is the set, A, of all points $\{(x, y) : y = \sqrt{x}$ where $A \subseteq \mathbb{R} \times \mathbb{R}$. we get the deriviative, $\frac{dy}{dx}$, but obviously it is meaningless at x$x = -1$ since that value does not yield a point in A.

7. Find the area of the region bounded by $x = -2, x = 3$, $y = 0$, and $h : \mathbb{R} \longrightarrow \mathbb{R}$ where $h(x) = \begin{cases} 3x + 15, & x < -1 \\ \\ x^3 - x^2, & x \geqslant -1 \end{cases}$

8. Find the volume of solid formed by rotating the region bounded by $y = x^3, y = -x$ and $x = 1$ around the y - axis.

9. Find the arc length of the curve $y = \sqrt{x^3}$ from (1, 1) to (4, 8).

10. Find the volume of the solid formed by rotating the region bounded by $y = x^3$, $x = 1, x = 2, y = 0$ around the x - axis.

11. Find the volume of the solid formed by rotating the region bounded by $y = x^3$, $x = 1, x = 2, y = 0$ around the line $x = 2$.

12. Find the volume of the solid formed by rotating the region bounded by $y = x^3$, $x = 1, x = 2, y = 0$ around the line $y = -2$.

13. Find the absolute maxima, minima, relative maxima, minima for $f(x) = (3x - 5)^2 \cdot (2x + 5)^3$ and $f : [-2, 5] \longrightarrow \mathbb{R}$.

14. Use Reimann Sums find the area of the region R which is defined as the planar figure enclosed by $y = x^2 + 3$, $x = 2$, $x = -1$, and $y = -x$.

15. Use differentials (*not the derivative incorrectly used Mr. F.*) to approximate $\sqrt[3]{499}$

Is there a word or phrase that is unfamiliar? Then look it up! This is university not kindergarten.

Do not come to class with statements or excuses such as:

"I don't know what to do."[3]

"I tried everything."[4]

"We didn't do this in (fill-in-the-blank of a course before Calculus II)."[5]

[3]Go remediate yourself. Get out your old Calculus I or pre-calculus book and if you don't have it with you then find a Calculus I or pre-calculus book (say, in the Library) and learn the Calculus or pre-calculus material.

[4]A less than truthful boast (who in the heck has really tried everything?).

[5]Go remediate yourself. Get out your old Calculus I or pre-calculus book and if you don't have it with you then find a Calculus I or pre-calculus book (say, in the Library) and learn the Calculus I or pre-calculus material.