Worksheet 8
GRAPHING USING CALCULUS - PART II
FINDING POINTS OF INFLECTION AND CONCAVITY USING CALCULUS - PART I

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Let the universe be $U = \mathbb{R} \times \mathbb{R}$ (the plane).

For each write **D. N. E.** for does not exist if an answer does not exist and explain why it does not exist.

Def'n 8.1: Let $f : D \longrightarrow C$ be a well defined function such that $D \subseteq \mathbb{R} \wedge C \subseteq \mathbb{R}$ and it be differentiable (first and second derivates) over D except (perhaps) at finitely many points of the function. Let $(a, b) \subseteq D$. f is **concave up over the segment** (a, b) *iff* $f^{''}(x) > 0 \quad \forall x \in (a, b)$

Def'n 8.2: Let $f : D \longrightarrow C$ be a well defined function such that $D \subseteq \mathbb{R} \wedge C \subseteq \mathbb{R}$ and it be differentiable (first and second derivates) over D except (perhaps) at finitely many points of the function. Let $(a, b) \subseteq D$. f is **concave down over the segment** (a, b) *iff* $f^{''}(x) < 0 \quad \forall x \in (a, b)$

Def'n 8.3: Let $f : D \longrightarrow C$ be a well defined function such that $D \subseteq \mathbb{R} \wedge C \subseteq \mathbb{R}$ and it be differentiable (first and second derivates) over D except (perhaps) at finitely many points of the function. The point $(p, f(p))$ is a **point of inflection** *iff* the concavity changes at the point, $f^{''}(p) = 0$ or $f^{''}(x)$ does not exist a p and $(p, f(p))$ exists (is a point of the function).

Exercise 8.1: Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a well defined function such that $f(x) = x^4 - x^3$.
Find the point(s) of inflection and find where f is concave up or concave down.

Exercise 8.2: Let $g : \mathbb{R} \longrightarrow \mathbb{R}$ be a well defined function such that $g(x) = x^4 + x^3$.
Find the point(s) of inflection and find where f is concave up or concave down.

Exercise 8.3: Let $h : D \longrightarrow \mathbb{R}$ be a well defined function such that $h(x) = x^4 + x^3$ where $D = [0, \infty)$.
Find the point(s) of inflection and find where f is concave up or concave down.

Exercise 8.4: Let $j : \mathbb{R} \longrightarrow \mathbb{R}$ be a well defined function such that $j(x) = \sqrt[3]{x}$.
Find the point(s) of inflection and find where f is concave up or concave down.

Exercise 8.5*: Let $p : \mathbb{R} \longrightarrow \mathbb{R}$ be a well defined function such that $p(x) = \frac{-3}{2} \cdot x^2 + \frac{1}{12} \cdot x^4 - \frac{1}{3} \cdot x^3$.
Find the point(s) of inflection and find where p is concave up or concave down.
Note: * designates a challenging problem ('hard').