Worksheet 12

RIEMANN SUMS, DEFINITE INTEGRALS, AND THE FUNDAMENTAL THEOREM OF CALCULUS

DR. M. P. M. M. MCLOUGHLIN

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Let $U = \mathbb{R} \times \mathbb{R}$

Theorem 12.1. *Fundamental Theorem of Calculus (part one). Let $f : D_1 \longrightarrow C_1$ such that $D_1 \subseteq \mathbb{R}$ and $C_1 \subseteq \mathbb{R}$. Let $a \in \mathbb{R}$ and $b \in \mathbb{R}$ such that $a < b$. Let $[a, b] \subseteq D_1$. Let f be continuous over (a, b); right continuous at a and left continuous at b.*[1] *Suppose there exists a function $F : D_2 \longrightarrow C_2$ such that $D_2 \subseteq \mathbb{R}$ and $C_2 \subseteq \mathbb{R}$ and $F'(x) = f(x)\ \forall\ x \in [a, b]$; then $\displaystyle\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a)$.*

Theorem 12.2. *Fundamental Theorem of Calculus (part two). Let $f : D_1 \longrightarrow C_1$ such that $D_1 \subseteq \mathbb{R}$ and $C_1 \subseteq \mathbb{R}$. Let $a \in \mathbb{R}$ and $b \in \mathbb{R}$ such that $a < b$. Let $[a, b] \subseteq D_1$. Let f be continuous over $[a, b]$. The function $g : [a, b] \longrightarrow \mathbb{R}$ such that $g(x) = \displaystyle\int_a^x f(t)dt$ is continuous on $[a, b]$, differentiable on (a, b) and moreover $g'(x) = f(x) \forall\ x \in (a, b)$.*

Exercise 12.1. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = 3 \cdot x + 4$. Let $g : (0, \infty) \longrightarrow \mathbb{R}$ such that $g(x) = \dfrac{1}{x}$. Let $x = 1$. Let $x = 3$. Let $y = 0$. Define T as the region bounded by $x = 3,\ x = 1,\ y = 0,\ \wedge\ f$. Draw T. Set up the Riemann Sum to find the area of T. Note you can not compleat the Riemann Sum; but, you can use the Fundamental Theorem of Calculus to evaluate the area. Please find the area of the region T.

Exercise 12.2. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = \sin x$. Let $x = \dfrac{\pi}{4}$. Let $x = \dfrac{\pi}{6}$. Let $y = 0$. Define R as the region bounded by these 4 curves. Draw R. Set up the Riemann Sum to find the area of R. Note you can not compleat the Riemann Sum; but, you can use the Fundamental Theorem of Calculus to evaluate the area. Please find the area of the region R.

Exercise 12.3. Evaluate $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\sec^2 x\right) dx$.

Exercise 12.4. Evaluate $\displaystyle\int_3^5 \left(3x^2 - 8\right)^2 dx$.

Exercise 12.5. Evaluate $\displaystyle\int_{\ln 3}^{\ln 5} (e^x) dx$.

Exercise 12.6. Evaluate $\displaystyle\int_{-1}^{3} (e^x + 2) dx$.

Exercise 12.7. Evaluate $\displaystyle\int_{e^2}^{e^3} \left(\frac{1}{x}\right) dx$.

Exercise 12.8. Evaluate $\displaystyle\int_2^3 \left(\frac{1}{x} + x\right) dx$.

Exercise 12.9. Evaluate $\displaystyle\int_1^3 \left(\frac{x^4 - 3x^3 + 2x}{x}\right) dx$.

[1] Recall we will shorten this to f is continuous over $[a, b]$.