MATH 181 CALCULUS I
DR. MCLOUGHLIN'S HANDY DANDY GUIDE TO DIFFERENTIATION MEANING HANDOUT 2 ¾
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Consider *f*(x) to be a well defined function in simplified form such that *f*: D $\longrightarrow \mathbb{R}$ where D $\subseteq \mathbb{R}$ You will notice that derivative of *f*(x) at (a, *f*(a)) is just a 'tad' more developed than limit or continuity and is built upon those concepts.

Definition 3.1: Let *f*: D $\longrightarrow \mathbb{R}$ be a well defined function.
Let x = a (or simply a) be a real number such that there is some positive real number b so that (a − b, a + b) $\subseteq$ D.
We say the function *f* has a derivative at a if and only if
it is the case that $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ exists.

Definition 3.2: Let *f*: D $\longrightarrow \mathbb{R}$ be a well defined function.
Let x = a (or simply a) be a real number such that there is some positive real number b so that (a − b, a + b) $\subseteq$ D.
We say the function *f* has a derivative at a if and only if
$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ exists.

Consequence of Definition 3.2: *f* ': A $\longrightarrow \mathbb{R}$ is a well defined function from the set A to the set $\mathbb{R}$ where A $\subseteq$ D. Remember the derivative is an equation that gives us the slope of the tangent line to a curve at any point.

1. Definition of the derivative

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

2. Examples:

A. *f*(x) = $9x^2$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{9(x+h)^2-9x^2}{h} = \lim_{h\to 0}\frac{9x^2+18xh+9h^2-9x^2}{h}$$

$$= \lim_{h\to 0}\frac{18xh+9h^2}{h} = \lim_{h\to 0} 18x+9h \xrightarrow{h\to 0} 18x$$

B. $P(q) = \dfrac{q+3}{q-2}$ $\qquad P'(q) = \lim\limits_{h\to 0}\dfrac{P(q+h)-P(q)}{h} =$

$$\lim_{h\to 0}\frac{\left(\dfrac{q+h+3}{q+h-2}-\dfrac{q+3}{q-2}\right)}{h} = \lim_{h\to 0}\frac{\left(\dfrac{(q+h+3)(q-2)-(q+3)(q+h-2)}{(q+h-2)(q-2)}\right)}{h}$$

$$= \lim_{h\to 0}\frac{\left(q^2+qh+3q-2q-2h-6-q^2-qh+2q-3q-3h+6\right)}{(q+h-2)(q-2)h} = \lim_{h\to 0}\frac{-5h}{(q+h-2)(q-2)h}$$

$$= \lim_{h\to 0}\frac{-5}{(q+h-2)(q-2)} \xrightarrow{h\to 0} \frac{-5}{(q-2)^2}$$

C. $h(z) = \sqrt{5z+7}$ $\qquad h'(z) = \dfrac{dh}{dz} = \lim\limits_{m\to 0}\dfrac{h(z+m)-h(z)}{m} =$

$$\lim_{m\to 0}\frac{\sqrt{5(z+m)+3}-\sqrt{5z+3}}{m} = \lim_{m\to 0}\frac{(\sqrt{5(z+m)+3}-\sqrt{5z+3})(\sqrt{5(z+m)+3}+\sqrt{5z+3})}{m(\sqrt{5(z+m)+3}+\sqrt{5z+3})}$$

$$= \lim_{m\to 0}\frac{5z+5m+3-5z-3}{h(\sqrt{5(z+m)+3}+\sqrt{5z+3})} = \lim_{m\to 0}\frac{5\cdot m}{m(\sqrt{5(z+m)+3}+\sqrt{5z+3})}$$

$$= \lim_{m\to 0}\frac{5}{(\sqrt{5z+5m+3}+\sqrt{5z+3})} \xrightarrow{m\to 0} \frac{5}{2\sqrt{5z+3}}$$

Average rate of change
Slope of secant line

$$\frac{R(x_2)-R(x_1)}{x_2-x_1}$$

Actual Change in revenue
Best when looking back

$$\frac{\Delta R}{\Delta x}$$

Instantaneous rate of change
Slope of tangent line

$$\lim_{h\to 0}\frac{R(x+h)-R(x)}{h}$$

Predicted change in revenue (marginal)
Best when looking ahead

$$\frac{dR}{dx}$$

Let's say we have $P(q) = \dfrac{q+3}{q-2}$ thus $P'(q) = \dfrac{-5}{(q-2)^2}$ where P is profit in hundreds of dollars and q is thousands of units. How much more (or less) profit will we make if we expand from 500 units to 1500 units?

Average rate of change
Actual Change in profit
Slope of secant line
$\dfrac{\Delta P}{\Delta q} = \dfrac{P(1.5)-P(.5)}{1.5-.5}$
Best when looking back (if we are making 500 units, we really *don't know* how much profit will be in 1500 units are made)
$-6\dfrac{2}{3}$ hundreds of dollars

Instantaneous rate of change
Predicted change in profit (marginal)
Slope of tangent line
$\dfrac{dP}{dq} = P'(.5) = \dfrac{-5}{(.5-2)^2}$
Best when looking ahead
$-2\dfrac{2}{9}$ hundreds of dollars