Worksheet Zero

Pre-calculus Concepts

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Due: Monday, the 24^{th} August 2015 at 12 Noon

Definition: $\mathbb{R} = \{x|$ where x is a point on the line$\}$.

Let $U = \mathbb{R}$ for the line and let $U = \mathbb{R} \times \mathbb{R}$ for the plane. U means the universe (Set Theory) which must be defined before a discussion of welements, points, sets, collections, etc.

Definition: $\mathbb{N}^* = \{0, 1, 2, 3, \ldots, (k-1), k, \ldots\}$

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Definition: $\mathbb{N}_k = \{1, 2, 3, \ldots, (k-1), k\}$

Definition: Let $a \in \mathbb{R}$, $b \in \mathbb{R}$ such that $a < b$.
Then a **segment** is (a, b) and an **interval** is $[a, b]$.
A **half-segment or half-interval** is $(a, b]$ or $[a, b)$.

Definition: $\mathbb{Z} = \{0, 1, -1, 2, -2, 3, , -3 \ldots\}$

Definition: $\mathbb{Q} = \{x | x = \frac{m}{n}, m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0\}$

Definition: $\mathbb{I} = \{x | x \in \mathbb{R} \wedge x \notin \mathbb{Q}\}$

Questions and Exercises

You may not use calculators, computers, etc. No help from any person other than yourself and from any notes other than your own. You may use other books: from the library, from a professor, etc. Use pencil only. All the necessary & sufficient steps for a solution should be shown - further, justification for each step should be provided. If an answer does not exist write D.N.E. (Does Not Exist) and explain why it does not exist.

Let $U = \mathbb{R}$

1 - 8 Determine if the following are true or false (no work need be done just think about these) and for all of these let x $\in \mathbb{R}$:

1. Claim: $0 < 1$.
2. Claim: $0.\overline{9} < 1$.
3. Claim: x$\cdot 0 = 0$.
4. Claim: x $\div$ 0 = 0
5. Claim: x $\div$ x = 1
6. Claim: 0 $\div$ x = 0
7. Claim: Let x $\in \mathbb{R}$,
$$\frac{x^3 - 1}{x - 1} = x^2$$
8. Claim: Let x $\in \mathbb{R}$,
$$\frac{x^3 - 1}{x - 1} = x^2 + x + 1$$

9. Let x, y $\in \mathbb{R}$ ∋ $(x + y) \neq 0$. Simplify $\frac{x^3 + y^3}{x + y}$
10. Let x, y $\in \mathbb{R}$ ∋ $x \neq y$. Simplify $\frac{x^3 - y^3}{x - y}$
11. Let x, y $\in \mathbb{R}$ ∋ $x \neq -y$. Simplify $\frac{x^2 + y^2}{x + y}$

12. Let x, y $\in \mathbb{R}$ ∋ $x \neq y$. Simplify $\dfrac{x^2 - y^2}{x - y}$

13. Let x, y $\in \mathbb{R}$ ∋ $(x + y) \neq 0, x \neq 0, y \neq 0, x \neq y$. Simplify $\dfrac{x^{-2}y^{-2}}{x^{-2} - y^{-2}}$

14. Let x, y $\in \mathbb{R}$. Find (simplify) the expression: $\left(x^3 + y^3\right) \cdot \left(x^2 + 2xy - y^2\right)$

Let $U = \mathbb{R} \times \mathbb{R}$

15 - 18 Determine if the following are true or false (no work need be done just think about these):

15. Claim: A function in the plane is invertible if and only if it is injective.[1]

16A. Claim: Let x $\in \mathbb{R}$, $\sin^2(x) + \cos^2(x) = 1 \quad \Rightarrow \quad \sin(x) + \cos(x) = 1$.

16B. Claim: Let $x, y \in \mathbb{R}$. It is the case that $\sin^2(x) + \cos^2(y) = 1$.

17. Claim: Let x $\in \mathbb{R}, f(x) = e^x \wedge g(x) = \ln(x)$. It is the case that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.

18. Claim: A function in the plane has a vertical asymptote at x = a if and only the function does not exist at x = a.

19. Consider

$$k(x) = \frac{x^3 + 3x^2 - 4x - 12}{x^2 - 3x + 2}$$

where $k : D \longrightarrow \mathbb{R}$ such that $D \subseteq \mathbb{R}$ where D is the largest subset of $\mathbb{R}$ that can be the domain of the function k. Find D.

20. Graph the following systematically. Beegin with $f(x) = x^2$ where $f : \mathbb{R} \longrightarrow \mathbb{R}$ and 'track' the point $(0, 0)$ through the different stages of the systematic graphing or begin with $g(x) = x^3$ where $g : \mathbb{R} \longrightarrow \mathbb{R}$ and 'track' the point $(0, 0)$ through the different stages of the systematic graphing (see systematic graphic review handout on my Math 181 web-page for a review or reminder of this material):

A. $f_A(x) = 3(x^2 - 5)$ B. $f_B(x) = -2(x + 1)^2 + 4$

C. $f_C(x) = \frac{1}{3}(x - 2)^2 - 1$ D. $f_D(x) = \frac{7}{2}(x)^2 + \pi$

E. $g_E(x) = 3(x^3 - 5)$ F. $g_F(x) = -2(x + 1)^3 + 4$

G. $g_H(x) = \frac{1}{3}(x - 2)^3 - 1$ H. $g_J(x) = \frac{7}{2}x^3 + \pi$

21. Consider $w : (4, \infty) \longrightarrow \mathbb{R}$ where

$$w(x) = \frac{x^3 - 3x^2 + 16x - 48}{x^2 - 7x + 12}$$

Find: $w(5), w(\frac{11}{2})$,w(0), $w(\frac{17}{4}), \wedge \quad w(\frac{401}{10})$

[1] Injective is the proper terminology for one-to-one function.

22. Consider $h : \mathbb{R} \longrightarrow \mathbb{R}$:

$$h(x) = \begin{cases} 3x + 15, & x < -4 \\ \frac{1}{x-1}, & x = -4 \\ 3x^2, & -4 < x \leqslant 1 \\ \frac{5x+7}{4}, & 1 < x < 5 \\ x^3 - x^2, & x \geqslant 5 \end{cases}$$

A. Find the domain of k. B. Find the codomain of k.
C. Graph k. D. Find the range of k.

23. Graph the following systematically. Begin with $k(x) = \sqrt{x}$ where $k : [0, \infty) \longrightarrow \mathbb{R}$ and 'track' the point $(0, 0)$ through the different stages of the systematic graphing; further note the domain and codomain of the new (final) function (see systematic graphic review handout on my Math 181 web-page for a review or reminder of this material):

A. $k_A(x) = \sqrt{x - 2}$ B. $k_B(x) = -\sqrt{x} + 4$

C. $k_C(x) = \frac{1}{3}\sqrt{x + 1} + 5$ D. $k_D(x) = \sqrt{x + 2}$

E. $k_E(x) = \sqrt{-x + 2}$ F. $k_F(x) = \sqrt{2 - x}$

24. Graph the following systematically. Begin with $j(x) = e^x$ where $j : \mathbb{R} \longrightarrow (0, \infty)$ and 'track' the point $(0, 1)$ through the different stages of the systematic graphing; further note the domain and **range** of the new (final) function (see systematic graphic review handout on my Math 181 web-page for a review or reminder of this material):

A. $j_A(x) = e^{(x-2)}$ B. $j_B(x) = e^x - 2$

C. $j_C(x) = e^{-x} + 5$ D. $j_D(x) = e^{x-2} + 3$

E. $j_E(x) = -3 \cdot e^{(x+1)}$ F. $j_F(x) = e^{(x-2)} + 3$

Is there a word or phrase that is unfamiliar? Then look it up! This is university not kindergarten.

Do not come to class with statements or excuses such as:

"I don't know what to do."[2]

"I tried everything."[3]

"We didn't do this in (fill-in-the-blank of a course before Calculus)."[4]

Finally note:
$\wedge$ means "and," $\vee$ means "or," '$\in$ means "is a member of the set," $\ni$ means "such that," and. means "implies."

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[2]Go remediate yourself. Get out your old pre-calculus book and if you don't have it with you then find a pre-calculus book (say, in the Library) and learn the pre-calculus material.

[3]A less than truthful boast (who in the heck has really tried everything?).

[4]Go remediate yourself. Get out your old pre-calculus book and if you don't have it with you then find a pre-calculus book (say, in the Library) and learn the pre-calculus material.