Worksheet $11\frac{1}{2}$
RIEMANN SUMS AND DEFINITE INTEGRALS
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Let $U = \mathbb{R} \times \mathbb{R}$

Exercise 11.1. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = x^2 + 1$. Let $g : \mathbb{R} \longrightarrow \mathbb{R}$ such that $g(x) = -3 \cdot x^2 + 1$ Let $x = 5$. Define R as the region bounded by $x = 5,\ f,\ \wedge\ g$. Draw R. Find the area of R using Riemann Sums.

Exercise 11.2. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = 3x + 4$. Let $g : \mathbb{R} \longrightarrow \mathbb{R}$ such that $g(x) = x^2$. Define S as the region bounded by $f,\ \wedge\ g$. Draw S. Find the area of S using Riemann Sums.

Exercise 11.3. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = 3 \cdot x + 4$. Let $g : (0, \infty) \longrightarrow \mathbb{R}$ such that $g(x) = \frac{1}{x}$. Let $x = 1$. Let $x = 3$. Let $y = 0$. Define T as the region bounded by $x = 3,\ x = 1,\ y = 0,\ \wedge\ f$. Draw T. Set up the Riemann Sum to find the area of T. Note you cannot compleat the Riemann Sum.

Definition 11.1. Let $f : D_1 \longrightarrow C_1$ such that $D_1 \subseteq \mathbb{R}$ and $C_1 \subseteq \mathbb{R}$. Let $a \in \mathbb{R}$ and $b \in \mathbb{R}$ such that $a < b$ and $[a, b] \subseteq D_1$. Let f be well defined $\forall\ x \in [a, b]$.

$\int_a^b f(x)dx$ is defined as $\lim_{n\to\infty} \left(\sum_{j=1}^{n} \left(f\left(a + \frac{j(b-a)}{n} \right) \cdot \left(\frac{b-a}{n} \right) \right) \right)$ **provided such limit exists**.

Definition 11.2. Let $f : D_1 \longrightarrow C_1$ such that $D_1 \subseteq \mathbb{R}$ and $C_1 \subseteq \mathbb{R}$. Let $a \in \mathbb{R}$ and $b \in \mathbb{R}$ such that $a < b$ and $[a, b] \subseteq D_1$. Let f be well defined $\forall\ x \in [a, b]$.

$\int_b^a f(x)dx$ is defined as $-\left(\int_a^b f(x)dx \right)$ provided such integral exists.

Exercise 11.4. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = x^2 - 3$. Let $x = -1$. Let $x = 3$. Evaluate $\int_{-1}^{3} f(x)dx = \int_{-1}^{3} \left(x^2 - 3\right)dx$

Exercise 11.5. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = x^2 - 3$. Evaluate $\int_3^3 f(x)dx = \int_3^3 \left(x^2 - 3\right)dx$

Exercise 11.6. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = x^2 - 3$. Evaluate $\int_1^3 \left(x^2 - 3\right)dx$

Exercise 11.7. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = x^2 - 3$. Evaluate $\int_1^2 \left(x^2 - 3\right)dx$

Exercise 11.8. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = x^2 - 3$. Evaluate $\int_2^3 \left(x^2 - 3\right)dx$

Theorem 11.1. *Letf* $: D_1 \longrightarrow C_1$ *such that* $D_1 \subseteq \mathbb{R}$ *and* $C_1 \subseteq \mathbb{R}$. *Let* $a \in \mathbb{R}$ *and* $b \in \mathbb{R}$ *such that* $a < b$ *and* $[a, b] \subseteq D_1$. *Let* $c \in [a, b]$. *It is the case that* $\int_c^c f(x)dx = 0$

Theorem 11.2. *Letf* $: D_1 \longrightarrow C_1$ *such that* $D_1 \subseteq \mathbb{R}$ *and* $C_1 \subseteq \mathbb{R}$. *Let* $a \in \mathbb{R}$ *and* $b \in \mathbb{R}$ *such that* $a < b$ *and* $[a, b] \subseteq D_1$. *Let* $c \in (a, b)$. *It is the case that* $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$