WORKSHEET 4
CALCULUS I
LIMITS OF INTERESTING EXPRESSIONS
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Reduce numerical results. If an answer *does not exist*, write **DNE** and explain why it does not exist!

1. Find $\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ if it exists

A. where $f(x)=x^3$ $\quad f:\mathbb{R}\longrightarrow\mathbb{R}$

B. where $f(x)=\sqrt{3x-7}$ $\quad f:[{}^7/_3\,,\infty)\longrightarrow\mathbb{R}$

C. where $f(x)=\frac{x^3}{x+1}$ $\quad f:(-\infty,-1)\cup(-1,\infty)\longrightarrow\mathbb{R}$

D. where $f(x)=\frac{3x+5}{5x-3}$ $\quad f:(-\infty,{}^3/_5)\cup({}^3/_5,\infty)\longrightarrow\mathbb{R}$

E. where $f(x)=4x^2-5x+8$ $\quad f:\mathbb{R}\longrightarrow\mathbb{R}$

2. Find $\lim_{x\to 3}\left(\frac{f(x)-f(3)}{x-3}\right)$ if it exists

A. where $f(x)=x^3$ $\quad f:\mathbb{R}\longrightarrow\mathbb{R}$

B. where $f(x)=\sqrt{3x-7}$ $\quad f:[{}^7/_3\,,\infty)\longrightarrow\mathbb{R}$

C. where $f(x)=\frac{x^3}{x+1}$ $\quad f:(-\infty,-1)\cup(-1,\infty)\longrightarrow\mathbb{R}$

D. where $f(x)=\frac{3x+5}{5x-3}$ $\quad f:(-\infty,{}^3/_5)\cup({}^3/_5,\infty)\longrightarrow\mathbb{R}$

E. where $f(x)=4x^2-5x+8$ $\quad f:\mathbb{R}\longrightarrow\mathbb{R}$

F. where $f(x)=|x-3|$ $\quad f:\mathbb{R}\longrightarrow\mathbb{R}$

3. Find the slope of the tangent line to $f(x) = \sqrt{3x - 7}$ $f : [{}^{7}/_{3}, \infty) \longrightarrow \mathbb{R}$ at the point $(3, \sqrt{2})$

4. Find the derivative of $f(x) = x^3 - 4x^2 + 8x - 9$ $f : \mathbb{R} \longrightarrow \mathbb{R}$ at $(1, -4)$ using the definition of the derivative (either form)

5. Find a function which provides the derivative of $f(x) = \dfrac{x^2 + 8x}{x^2 - 2x}$ $f : D \longrightarrow \mathbb{R}$ at the points (x, y) using the definition of the derivative where $D = (-\infty, 0) \cup (0, 2) \cup (2, \infty)$

The function f has domain $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$ -- where is this new function, which we call $f'(x)$, well defined (what is its domain that is the maximal subset of D (for what x-values))?