Handout VI

Derivatives of Some Transcendental & Special Functions

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Let our universe be $\mathbb{R} \times \mathbb{R}$ which is the Cartesian plane. Let $D_i \subseteq \mathbb{R}$ be the appropriate standard domain for a function and $C_i \subseteq \mathbb{R}$ be the appropriate standard codomain for a function discussed in this handout $\forall i \in \mathbb{N}$.
You will notice that derivative of a function $f(x)$ over a set $A \subseteq \mathbb{R}$ is wonderful for many types of functions.

The trigonometric functions.

$g_1 : \mathbb{R} \to \mathbb{R} \ni g_1(x) = \sin(x) \qquad g_1(x) = \sin(x) \implies g_1'(x) = \cos(x)$
$g_2 : \mathbb{R} \to \mathbb{R} \ni g_2(x) = \cos(x) \qquad g_2(x) = \cos(x) \implies g_2'(x) = -\sin(x)$
$g_3 : D_3 \to \mathbb{R} \ni g_3(x) = \tan(x) \qquad g_3(x) = \tan(x) \implies g_3'(x) = \sec^2(x)$
$g_4 : D_4 \to \mathbb{R} \ni g_4(x) = \cot(x) \qquad g_4(x) = \cot(x) \implies g_4'(x) = -\csc^2(x)$
$g_5 : D_5 \to \mathbb{R} \ni g_5(x) = \sec(x) \qquad g_5(x) = \sec(x) \implies g_5'(x) = \sec(x) \cdot \tan(x)$
$g_6 : D_6 \to \mathbb{R} \ni g_6(x) = \csc(x) \qquad g_6(x) = \csc(x) \implies g_6'(x) = -(\csc(x) \cdot \cot(x))$

The trigonometric functions of another function (by the chain rule, obviously).

$g_7 : D_7 \to \mathbb{R} \ni g_7(x) = \sin(f(x)) \qquad g_7(x) = \sin(f(x)) \implies g_7'(x) = \cos(f(x)) \cdot f'(x) = f'(x) \cdot \cos(f(x))$
$g_8 : D_8 \to \mathbb{R} \ni g_8(x) = \cos(f(x)) \qquad g_8(x) = \cos(f(x)) \implies g_8'(x) = -\sin(f(x)) \cdot f'(x) = -f'(x) \cdot \sin(f(x))$
$g_9 : D_9 \to \mathbb{R} \ni g_9(x) = \tan(f(x)) \qquad g_9(x) = \tan(f(x)) \implies$
$g_9'(x) = \sec^2(f(x)) \cdot f'(x) = f'(x) \cdot \sec^2(f(x))$
$g_{10} : D_{10} \to \mathbb{R} \ni g_{10}(x) = \cot(f(x)) \qquad g_{10}(x) = \cot(f(x)) \implies$
$g_{10}'(x) = -\csc^2(f(x)) \cdot f'(x) = -f'(x) \cdot \csc^2(f(x))$
$g_{11} : D_{11} \to \mathbb{R} \ni g_{11}(x) = \sec(f(x)) \qquad g_{11}(x) = \sec(f(x)) \implies$
$g_{11}'(x) = \sec(f(x)) \cdot \tan(f(x)) \cdot f'(x) = f'(x) \cdot \sec(f(x)) \cdot \tan(f(x))$
$g_{12} : D_{12} \to \mathbb{R} \ni g_{12}(x) = \csc(f(x)) \qquad g_{12}(x) = \csc(f(x)) \implies$
$g_{12}'(x) = -\csc(f(x)) \cdot \cot(f(x))'(x) = -f'(x) \cdot \csc(f(x)) \cdot \cot(f(x))$

The logarithmic functions.

$g_{13} : (0, \infty) \to \mathbb{R} \ni g_{13}(x) = \ln(x) \qquad g_{13}(x) = \ln(x) \implies g_{13}'(x) = \dfrac{1}{x}$

$g_{14} : (0, \infty) \to \mathbb{R} \ni a \in \mathbb{R} \wedge a > 0 \ni g_{14}(x) = \log_a(x) \qquad g_{14}(x) = \log_a(x) \implies g_{14}'(x) = \dfrac{1}{x \cdot \ln(a)}$

$g_{15} : D_{15} \to \mathbb{R} \ni g_{15}(x) = \ln(f(x)) \qquad g_{15}(x) = \ln(f(x)) \implies g_{15}'(x) = \dfrac{f'(x)}{(f(x))}$

$g_{16} : D_{16} \to \mathbb{R} \ni a \in \mathbb{R} \wedge a > 0 \ni g_{16}(x) = \log_a(f(x)) \qquad g_{16}(x) = \log_a(f(x)) \implies g_{16}'(x) = \dfrac{f'(x)}{(f(x)) \cdot \ln(a)}$

The exponential functions.

$g_{26} : \mathbb{R} \to (0, \infty) \ni \quad g_{26}(x) = e^x \qquad g_{26}(x) = e^x \implies g'_{26}(x) = e^x$

$g_{27} : D_{27} \to C_{27} \ni \quad g_{27}(x) = e^{(f(x))} \qquad g_{27}(x) = e^{(f(x))} \implies g'_{27}(x) = e^{(f(x))} \cdot f'(x)$

$g_{28} : \mathbb{R} \to (0, \infty) \ni \quad g_{28}(x) = a^x \ni a \in \mathbb{R} \ni a > 0 \qquad g_{28}(x) = a^x \implies g'_{28}(x) = a^x \cdot \ln a$

$g_{29} : D_{29} \to C_{29} \ni \quad g_{29}(x) = a^{(f(x))} \ni a \in \mathbb{R} \ni a > 0 \quad g_{29}(x) = a^{(f(x))} \implies g'_{29}(x) = a^{(f(x))} \cdot f'(x) \cdot \ln a$

The three inverse trigonometric functions of note.

$g_{17} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \ni g_{17}(x) = \arcsin(x)$

$$g_{17}(x) = \arcsin(x) \implies g'_{17}(x) = \frac{1}{\sqrt{1 - x^2}}$$

$g_{18} : D_{18} \to C_{18} \ni g_{18}(x) = \arcsin(f(x))$

$$g_{18}(x) = \arcsin(f(x)) \implies g'_{18}(x) = \frac{f'(x)}{\sqrt{1 - (f(x))^2}}$$

$g_{19} : (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \ni g_{19}(x) = \arctan(x)$

$$g_{19}(x) = \arctan(x) \implies g'_{19}(x) = \frac{1}{1 + x^2}$$

$g_{20} : D_{20} \to C_{20} \ni g_{20}(x) = \arctan(f(x))$

$$g_{20}(x) = \arctan(f(x)) \implies g'_{20}(x) = \frac{f'(x)}{1 + (f(x))^2}$$

$g_{21} : D_{21} \to C_{21} \ni g_{21}(x) = arcsec(x)$

$$g_{21}(x) = arcsec(x) \implies g'_{21}(x) = \frac{1}{|x| \cdot \sqrt{x^2 - 1}}$$

$g_{22} : D_{22} \to C_{22} \ni g_{22}(x) = arcsec(f(x))$

$$g_{22}(x) = arcsec(f(x)) \implies g'_{22}(x) = \frac{f'(x)}{|(f(x))| \cdot \sqrt{(f(x))^2 - 1}}$$

The three hyperbolic 'trigonometric' functions of note.

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$
$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$
$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$g_{23} : \mathbb{R} \to \mathbb{R} \ni g_{23}(x) = \sinh(x) \qquad g_{23}(x) = \sinh(x) \implies g'_{23}(x) = \cosh(x)$

$g_{24} : \mathbb{R} \to C_{24} \ni g_{24}(x) = \cosh(x) \qquad g_{24}(x) = \cosh(x) \implies g'_{24}(x) = \sinh(x)$

$g_{25} : \mathbb{R} \to C_{25} \ni g_{25}(x) = \tanh(x) \qquad g_{25}(x) = \tanh(x) \implies g'_{25}(x) =^2 (x)$