Worksheet I

Pre-calculus Concepts

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Spring 2011

Definition: $\mathbb{R} = \{x|$ where x is a point on the line$\}$.

Let $U = \mathbb{R}$ for the line and let $U = \mathbb{R} \times \mathbb{R}$ for the plane.

Definition: $\mathbb{N}^* = \{0, 1, 2, 3, \ldots, (k-1), k, \ldots\}$

Definition: $\mathbb{N} = \{1, 2, 3, \ldots, (k-1), k, \ldots\}$

Definition: $\mathbb{N}_k^* = \{0, 1, 2, 3, \ldots, (k-1), k\}$

Definition: $\mathbb{N}_k = \{1, 2, 3, \ldots, (k-1), k\}$

Definition: Let $a \in \mathbb{R}$, $b \in \mathbb{R}$ such that $a < b$.
Then a **segment** is $(a, b) = \{x \,|a < x < b\}$, the **interval** is $[a, b] = \{x \,|a \leq x \leq b\}$,
a **half-segment or half-interval** is $(a, b] = \{x \,|a < x \leq b\}$ or $[a, b) = \{x \,|a \leq x < b\}$.

Definition: $\mathbb{Z} = \{0, 1, -1, 2, -2, 3, , -3 \ldots\}$

Definition: $\mathbb{Q} = \{x | x = \frac{m}{n}, m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0\}$

Definition: $\mathbb{I} = \{x | x \in \mathbb{R} \wedge x \notin \mathbb{Q}\}$

Questions and Exercises

You may not use calculators, computers, etc. No help from any person other than yourself and from any notes other than your own. You may use other books: from the library, a professor, etc. Use pencil only. All the necessary sufficient steps for a solution should be shown - further, justification for each step should be provided. If an answer does not exist write D.N.E. (Does Not Exist) and explain why it does not exist.

Let $U = \mathbb{R}$

Problems 1 - 6 Determine if the following are true or false (no work need be done just think about these) and for all of these let x $\in \mathbb{R}$:

1. Claim: $0 < 1$.
2. Claim: $0.\overline{9} < 1$.
3. Claim: x$\cdot 0 = 0$.
4. Claim: x $\div$ 0 = 0
5. Claim: Let x $\in \mathbb{R}$, $\dfrac{x^3 - 1}{x - 1} = x^2$
6. Claim: Let x $\in \mathbb{R}$, $\dfrac{x^3 - 1}{x - 1} = x^2 + x + 1$
7. Claim: Let x, y $\in \mathbb{R} \ni x \neq 0 \wedge y \neq 0$ $\dfrac{x^{-2}y^{-2}}{x^{-2} - y^{-2}} = \dfrac{x^2 - y^2}{x^2 y^2}$ Show it is true or false.
8. Let x, y $\in \mathbb{R}$. Find $\left(x^3 + y^3\right) \cdot \left(x^2 + 2xy - y^2\right)$
9. Let x, y $\in \mathbb{R} \ni \{x + y) \neq 0$. Simplify $\dfrac{x^3 + y^3}{x + y}$
10. Let x, y $\in \mathbb{R} \ni x \neq y$. Simplify $\dfrac{x^3 - y^3}{x - y}$
11. Let x, y $\in \mathbb{R} \ni \{x + y) \neq 0$. Simplify $\dfrac{x^2 + y^2}{x + y}$
12. Let x, y $\in \mathbb{R} \ni x \neq y$. Simplify $\dfrac{x^2 - y^2}{x - y}$
13. Consider $U = \mathbb{R} \times \mathbb{R}$ and $k(x) = \dfrac{x^3 + 3x^2 - 4x - 12}{x^2 - 3x + 2}$ $\ni k : D \longrightarrow \mathbb{R}$ such that $D \subseteq \mathbb{R}$ where D is the largest subset of $\mathbb{R}$ that can be the domain of the function k. Find D.

Problems 14 - 17 Determine if the following are true or false (no work need be done just think about these):

14. Claim: A function in the plane is invertible if and only if it is injective.
15. Claim: Let x $\in \mathbb{R}$, $\sin^2(x) + \cos^2(x) = 1 \quad \Rightarrow \quad \sin(x) + \cos(x) = 1$.
16. Claim: Let x $\in \mathbb{R}$, $f(x) = e^x \wedge g(x) = \ln(x)$. It is the case that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.
17. Claim: A function in the plane has a vertical asymptote at x = a if and only the function does not exist at x = a.

18. Graph the following systematically begin with $f(x) = x^2$ where $f : \mathbb{R} \longrightarrow \mathbb{R}$ and 'track' the point $(0, 0)$ through the different stages of the systematic graphing or begin with $g(x) = x^3$ where $g : \mathbb{R} \longrightarrow \mathbb{R}$ and 'track' the point $(0, 0)$ through the different stages of the systematic graphing:
A. $f_A(x) = 3(x^2 - 5)$
B. $f_B(x) = -2(x + 1)^2 + 4$
C. $f_C(x) = \frac{1}{3}(x - 2)^2 - 1$
D. $f_C(x) = \frac{7}{2}(x)^2 + \pi$
E. $g_E(x) = 3(x^3 - 5)$
F. $g_F(x) = -2(x + 1)^3 + 4$
G. $g_H(x) = \frac{1}{3}(x - 2)^3 - 1$
H. $g_J(x) = \frac{7}{2}x^3 + \pi$
19. Consider $h : \mathbb{R} \longrightarrow \mathbb{R}$:

$$h(x) = \begin{cases} 3x + 15, & x < -4 \\ \frac{1}{x-1}, & x = -4 \\ 3x^2, & -4 < x \leqslant 1 \\ \frac{5x+7}{4}, & 1 < x < 5 \\ x^3 - x^2, & x \geqslant 5 \end{cases}$$

A. Find the domain of k. B. Find the codomain of k.
C. Graph k. D. Find the range of k.

Is there a word, phrase, or some notation that is unfamiliar? Then look it up! Do not come to class with statements or excuses such as:

"I don't know what to do."

"I tried everything."[1]

"We didn't do this in (fill-in-the-blank of a course before Calculus)."[2]

You may wish to download some backgroud material at:
http://faculty.kutztown.edu/mcloughl/171handouts.html

[1] But, did and have nothing.

[2] Go remediate yourself. Get out your old pre-calculus book and if you don't have it with you then find a pre-calculus book (say, in the Library) and learn the pre-calculus material.