

L'HÔPITAL'S THEOREM
FINDING LIMITS:
TO USE L'HÔPITAL'S RULE OR NOT,
THAT IS THE QUESTION.

Let the $U = \mathbb{R}$ and the codomain of each function on this worksheet be \mathbb{R}

Find the following limits and use L'Hôpital's rule (where appropriate). Justify the methodology used for finding the limit (bulldozer, algebraic manipulation, plug-in method, L'Hôpital's rule, or our limit theorems (handout 1.5):

Exercise 4 – 1: Let $U = \mathbb{R}$. Define c as $c(x) = \frac{x^2}{e^x} \forall x \in \mathbb{R}$. Find $\lim_{x \rightarrow 0} (c(x)) = \lim_{x \rightarrow 0} \left(\frac{x^2}{e^x} \right)$

Exercise 4 – 2: Let $U = \mathbb{R}$. Define c as $c(x) = \frac{x^2}{e^x} \forall x \in \mathbb{R}$ Find $\lim_{x \rightarrow \infty} (c(x))$

Exercise 4 – 3: Let $U = \mathbb{R}$. Define g as $g(x) = \frac{x^2}{\sin(x)} \forall x \in \mathbb{R} \setminus \{x: x \neq k\pi, k \in \mathbb{Z}\}$.

Find $\lim_{x \rightarrow 0} (g(x))$

Exercise 4 – 4: Let $U = \mathbb{R}$. Define g as $g(x) = \frac{x^2}{\sin(x)} \forall x \in \mathbb{R} \setminus \{x: x \neq k\pi, k \in \mathbb{Z}\}$.

Find $\lim_{x \rightarrow \infty} (g(x))$

Exercise 4 – 5: Let $U = \mathbb{R}$. Define f as $f(x) = x \cdot \ln(x) \forall x \in (0, \infty)$ Find $\lim_{x \rightarrow 0} (f(x))$

Exercise 4 – 6: Let $U = \mathbb{R}$. Define f as $f(x) = x \cdot \ln(x) \forall x \in (0, \infty)$ Find $\lim_{x \rightarrow \infty} (f(x))$

Exercise 4 – 7: Let $U = \mathbb{R}$. Let $f: D \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{x^2 + 2x - 8}{x^2 + x - 6}$ where

$D = (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$ Find $\lim_{x \rightarrow -3} (f(x))$

Exercise 4 – 8: Let $U = \mathbb{R}$. Let $f: D \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{x^2 + 2x - 8}{x^2 + x - 6}$ where

$D = (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$ Find $\lim_{x \rightarrow 2} (f(x))$

Exercise 4 – 9: Let $U = \mathbb{R}$. Let $f: D \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{x^2 + 2x - 8}{x^2 + x - 6}$ where

$D = (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$ Find $\lim_{x \rightarrow 1} (f(x))$

Exercise 4 – 10: Let $U = \mathbb{R}$. Let $l: (-\infty, -3) \cup (-3, \infty) \longrightarrow \mathbb{R}$ be defined by $l(x) = \frac{4x^2 + x - 7}{3x^3 + x}$

Find $\lim_{x \longrightarrow \infty} l(x)$

Exercise 4 – 11: Let $U = \mathbb{R}$. Let $l: (-\infty, -3) \cup (-3, \infty) \longrightarrow \mathbb{R}$ be defined by $l(x) = \frac{4x^2 + x - 7}{3x^3 + x}$

Find $\lim_{x \longrightarrow 3} l(x)$

Exercise 4 – 12: Let $U = \mathbb{R}$. Let $l: (-\infty, -3) \cup (-3, \infty) \longrightarrow \mathbb{R}$ be defined by $l(x) = \frac{4x^2 + x - 7}{3x^3 + x}$

Find $\lim_{x \longrightarrow -3} l(x)$

Exercise 4 – 13: Let $U = \mathbb{R}$. Define $k: D \longrightarrow \mathbb{R}$ by $k(x) = x \cdot \tan(x)$ where $D = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Find $\lim_{x \longrightarrow 0} (f(x))$

Exercise 4 – 14: Let $U = \mathbb{R}$. Define $k: D \longrightarrow \mathbb{R}$ by $k(x) = x \cdot \tan(x)$ where $D = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Find $\lim_{x \longrightarrow \frac{\pi}{2}} (f(x))$