MATH 140 DR. MCLOUGHLIN'S CLASS
SOME SAMPLING DISTRIBUTION RESULTS
HANDOUT V

Let n, p, α, β, σ, μ, γ, θ, λ, c be constants.

Theorem 1 (DeMoivre - Laplace): Let X ~ Bin (x, n, p). Let $Y = \dfrac{X - np}{\sqrt{np(1-p)}}$.

As $n \longrightarrow \infty$, it is the case that $Y \longrightarrow Z$ where Z ~ Nor (z, 0, 1).

Theorem 2: Let X ~ Nor (x, μ,σ). Let $Z = \dfrac{X - \mu}{\sigma}$ it is the case that Z ~ Nor (z, 0, 1).

Theorem 3: Let X ~ Nor (x, μ_X,σ_X). Let $X_1, X_2, \ldots, X_n$ be independent, identically distributed random variables from X ~ Nor (x, μ_X,σ_X). Let Y = $\bar{X}$ it is the case that
$E[Y] = E[\bar{X}] = \mu_X$. So, $\mu_{\bar{X}} = \mu_X$.

Theorem 4: Let X ~ Nor (x, μ_X,σ_X). Let $X_1, X_2, \ldots, X_n$ be independent, identically distributed random variables from X ~ Nor (x, μ_X,σ_X). Let Y = $\bar{X}$ it is the case that
$Var[Y] = \dfrac{\sigma_X^2}{n}$. So, $\sigma_{\bar{X}}^2 = \dfrac{\sigma_X^2}{n}$.

Theorem 5: Let X ~ Nor (x, μ_X,σ_X). Let $X_1, X_2, \ldots, X_n$ be independent, identically distributed random variables from X ~ Nor (x, μ_X,σ_X). Let $Y = s^2 = \sum_{i=1}^{n} \dfrac{(X - \bar{X})^2}{n-1} = \dfrac{1}{n-1}\sum_{i=1}^{n}(X - \bar{X})^2$.

$E[Y] = \sigma_X^2$. So, $E[s^2] = \sigma_X^2$.

Theorem 6: Let X ~ Nor (x, μ,σ). Let $X_1, X_2, \ldots, X_n$ be independent, identically distributed random variables from X ~ Nor (x, μ,σ) Let Y = $\bar{X}$
it is the case that Y ~ Nor (y, μ, $\dfrac{\sigma}{\sqrt{n}}$).

Theorem 7: Let $X_1, X_2, \ldots, X_n$ be independent, identically distributed random variables from X ~ Nor (x, μ, σ). Let $Y = \dfrac{(n-1)s^2}{\sigma^2}$ it is the case that $Y \sim \chi^2_{(n-1)}$.

Theorem 8 (Gossett): Let Z ~ Nor(z, 0, 1), $U \sim \chi^2_m$, and Z and U be independent.
Let $W = \dfrac{Z}{\sqrt{U/m}}$ it is the case that W ~ t_m (t with m degrees of freedom or t with *df* = m).

Theorem 9: Let X ~ Nor (x, μ_X,σ_X). Let $X_1, X_2, \ldots, X_n$ be independent, identically distributed random variables from X ~ Nor (x, μ_X,σ_X)
it is the case that $\dfrac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}}$ ~ Nor (z, 0,1) .

Theorem 10 (Gossett): Let X ~ Nor (x, μ,σ). Let $X_1, X_2, \ldots, X_n$ be independent, identically distributed random variables from X ~ Nor (x, μ,σ)
it is the case that $\dfrac{\bar{X}-\mu}{s/\sqrt{n}} \sim t_{(n-1)}$.

Theorem 11 (Fisher – Snedecor): Let U ~ χ^2_n and V ~ χ^2_m and U and V be independent. Let Y = $\dfrac{U/n}{V/m}$ it is the case that Y ~ $F_{n, m}$.

Theorem 12: Let U ~ χ^2_n and V ~ χ^2_m and U and V be independent. Let Y = $\dfrac{U/n}{V/m}$.

Let W = $\dfrac{1}{Y}$ it is the case that W ~ $F_{m, n}$.

Theorem 13: Let $X_{11}, X_{12}, \ldots, X_{1n_1}$ be independent, identically distributed random variables from X_1 ~ Nor (x_1, μ_1,σ_1). Let $X_{21}, X_{22}, \ldots, X_{2n_2}$ be independent, identically distributed random variables from X_2 ~ Nor (x_2, μ_2,σ_2).
Let Y = $\dfrac{s_1^2\sigma_2^2}{s_2^2\sigma_1^2}$ it is the case that Y ~ F_{n_1-1,n_2-1} .

Theorem 14 (Central Limit Theorem / Law of Large Numbers) :
Let X ~ $f_X(x)$. Let E[X] = μ_x and Var [X] = σ^2_x be constant.

(1) As $n \longrightarrow \infty$, it is the case that $X \longrightarrow Y_1$ where Y_1 ~ Nor(y_1, n·E[X], $\sqrt{n}\cdot$SD[X]).
(2) Let $Y_2 = \bar{X}$
As $n \longrightarrow \infty$, it is the case that $Y_2 \longrightarrow Y_3$ where Y_3 ~ Nor(y_3, E[X], $\dfrac{SD[X]}{\sqrt{n}}$).

Definition 1: Let X ~ $f_X(x)$. Let α, β, σ, μ, γ, λ, . . ., θ be (possible) parametres for $f_X(x)$.
We say $\hat{\alpha}$ is an estimator of α, $\hat{\beta}$ is an estimator for β, . . . , $\hat{\theta}$ is an estimator for θ.

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