MATH 100
DR. MCLOUGHLIN'S HANDY DANDY GRAPHING GUIDE USING SYSTEMATIC OR POSITIVE NEGATIVE ANALYSIS PART II

$y = A\,f(B(x - C)) + D$

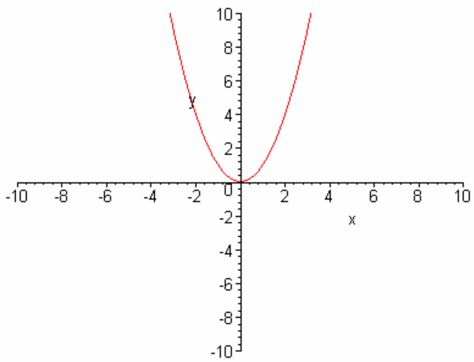
A stretches or contracts and flips along the y – axis
B stretches or contracts and flips along the x – axis
C shifts left or right along the x – axis
D shifts down or up along the y – axis

<u>Example 1:</u>

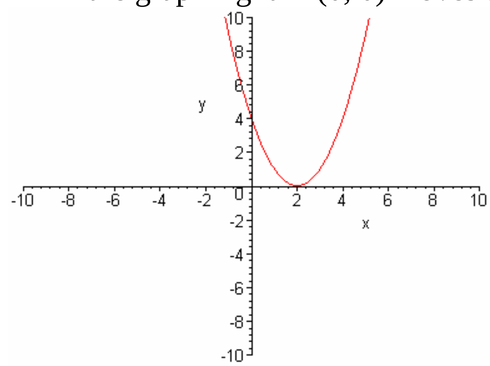
Systematically graph $y = 3(x-2)^2 - 4$

Note the domain is ℝ and the codomain is ℝ.

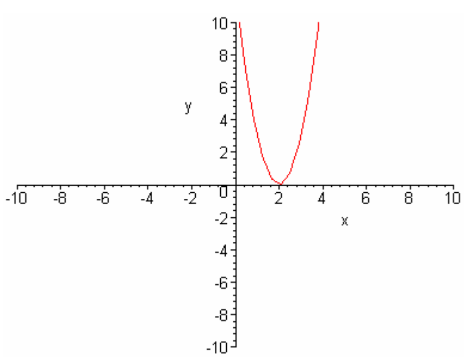
First graph $y = x^2$ notice the point (0, 0)



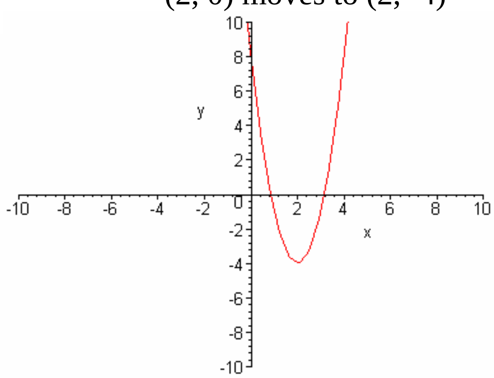
then $y = (x-2)^2$ which shifts the graph right 2 (0, 0) moves to (2, 0)



then $y = 3(x-2)^2$ which stretches the graph with respect to y
(2, 0) doesn't move



and finally $y = 3(x-2)^2 - 4$ which shifts the graph down 4
(2, 0) moves to (2, -4)



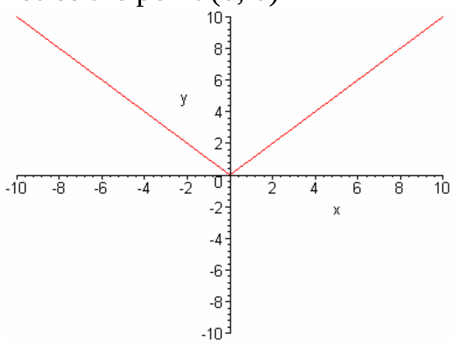
Now the domain is ℝ, the codomain is ℝ, and the range is $[-4, \infty)$

<u>Example 2:</u>

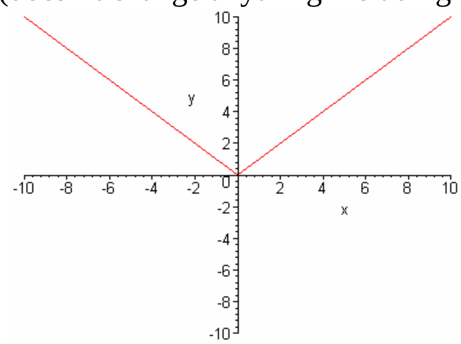
Systematically graph $y = \frac{1}{2}\left|-x+1\right| + \pi$

Note the domain is $\mathbb{R}$ and the codomain is $\mathbb{R}$

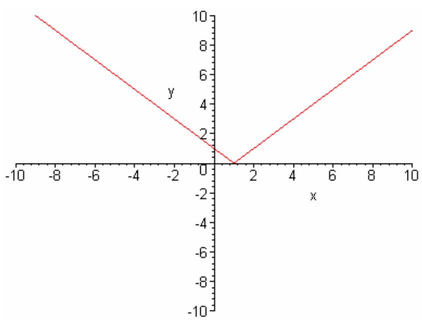
First graph y = | x |
notice the point (0, 0)



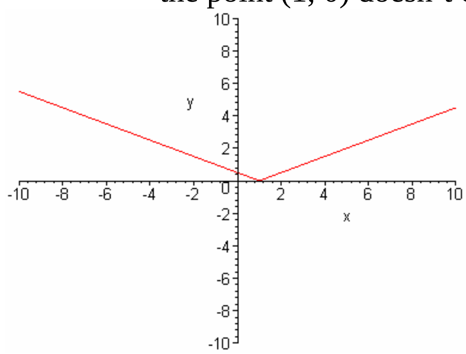
then y = | -x | which reflects it across the y-axis
(doesn't change anything including the point (0, 0))



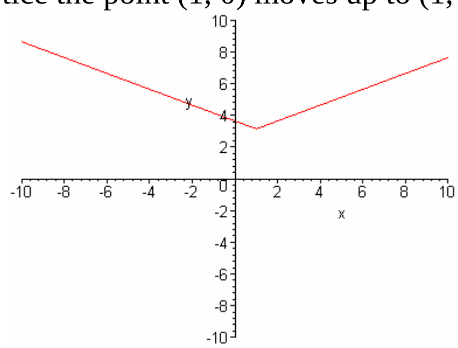
then $y = \left|-x+1\right|$ which shifts the graph right one
(0, 0) moves to (1, 0)



then $y = \frac{1}{2}\left|-x+1\right|$ which contracts the graph with respect to the y-axis ("squeezes it down some')
the point (1, 0) doesn't change



and finally, $y = \frac{1}{2}\left|-x+1\right| + \pi$ which shifts the graph up π.
notice the point (1, 0) moves up to (1, π).



Note the domain is $\mathbb{R}$, the codomain is $\mathbb{R}$, and the range is $[\pi, \infty)$

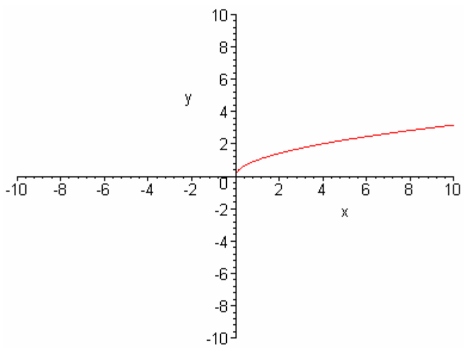
Example 3:

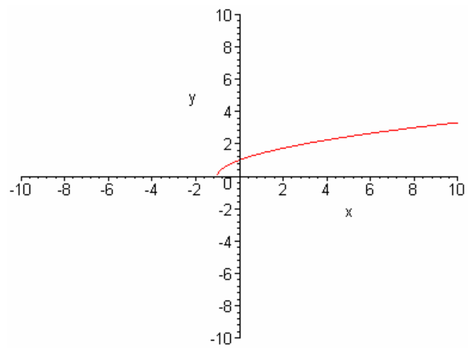
Systematically graph $y = \frac{-5}{3}\sqrt{x+1}$

Note the domain is [-1, ∞) and the codomain is ℝ

First graph $y = \sqrt{x}$ (the domain is [0, ∞)) notice the point (0, 0)

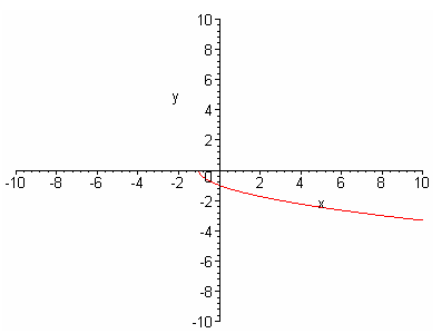
then $y = \sqrt{x+1}$ (the domain is now [-1, ∞)) which shifts it left 1 and (0, 0) moves to (-1, 0)

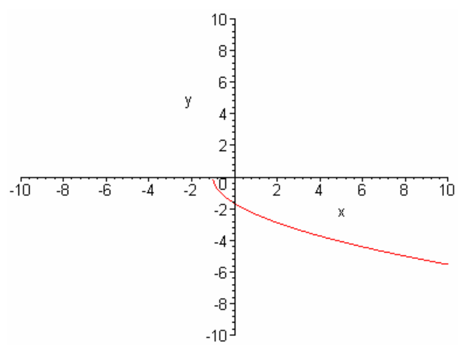




then $y = -\sqrt{x+1}$ which reflects it across the x-axis
(-1, 0) doesn't move

then $y = \frac{-5}{3}\sqrt{x+1}$ which stretches the graph with respect to the y-axis
(-1, 0) doesn't move





Note the domain is [-1, ∞), the codomain is ℝ, and the range is $(-\infty, 0]$.

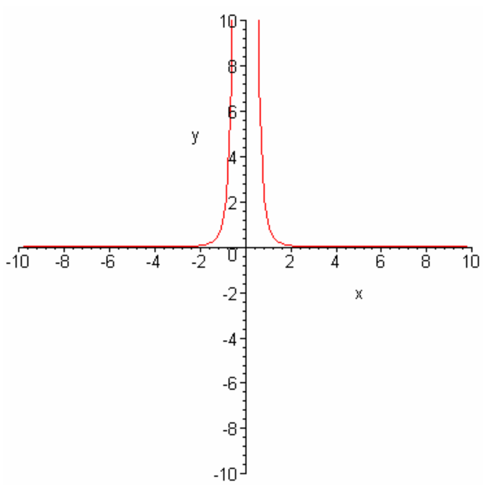
<u>Example 4:</u>

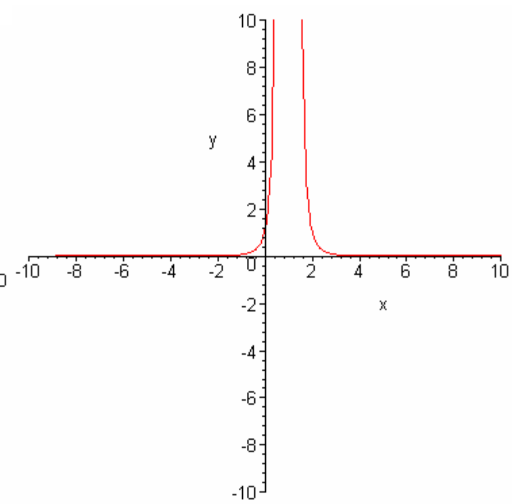
Systematically graph $y = 2(x-1)^{-4} + 3$

Note the domain is $(-\infty, 1) \cup (1, \infty)$ and the codomain is $\mathbb{R}$

First graph $y = x^{-4} = \frac{1}{x^4}$
(the domain is $(-\infty, 0) \cup (0, \infty)$)
notice there isn't the point (0, 0)
on this graph – - there is a vertical asymptote x = 0

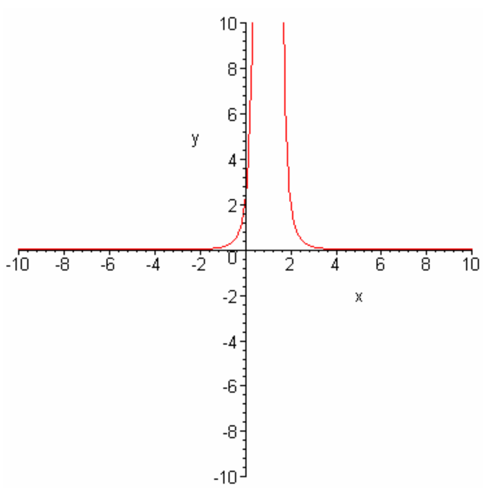
then $y = (x-1)^{-4}$
(the domain is now $(-\infty, 1) \cup (1, \infty)$)
which shifts it right 1
the vert. asy. x = 0 moves to x = 1.

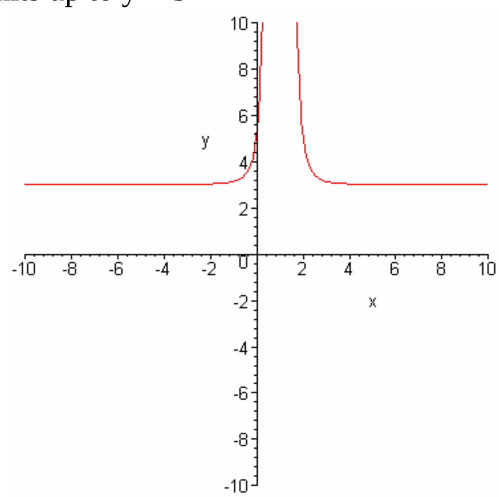




Now graph $y = 2(x-1)^{-4}$
which stretches it with respect
to the y-axis (hardly noticeable)
the vert. asy. is x = 1

and, finally, graph $y = 2(x-1)^{-4} + 3$
which shifts it up 3
so the horizontal asymptote y = 0
shits up to y = 3





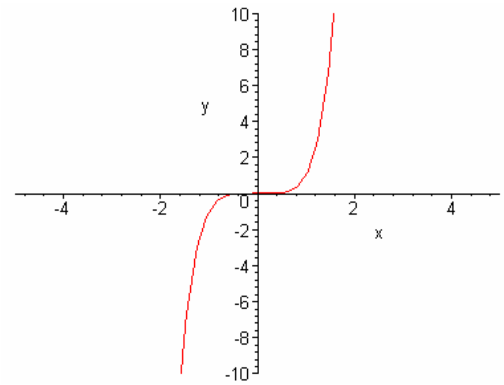
Note the domain is $(-\infty, 1) \cup (1, \infty)$, the codomain is $\mathbb{R}$, and the range is $(3, \infty)$.

Example 5:

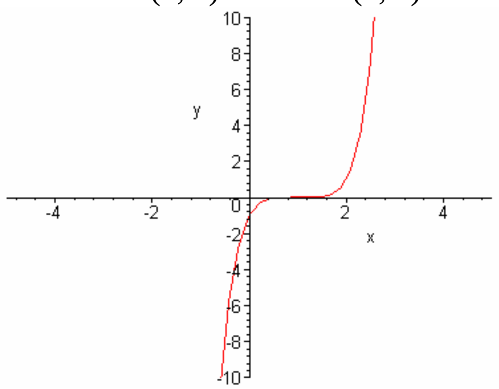
Systematically graph $y = 2(x-1)^5 + 3$

Note the domain is $\mathbb{R}$ and the codomain is $\mathbb{R}$

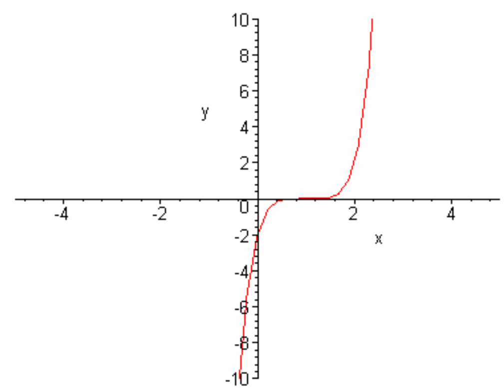
First graph $y = x^5$
notice the point (0, 0)



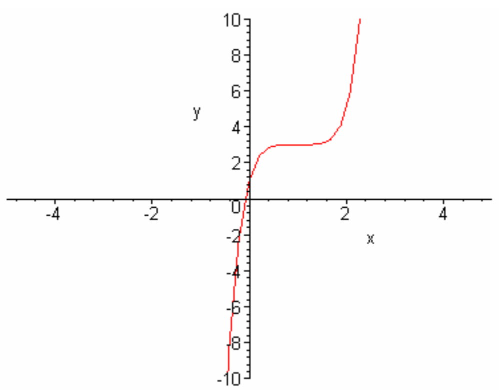
then $y = (x-1)^5$
which shifts it right 1
(0, 0) moves to (1, 0)



Now graph $y = 2(x-1)^5$
which stretches it with respect
to the y-axis
(1, 0) doesn't move



and, finally, graph $y = 2(x-1)^5 + 3$
which shifts it up 3
(1, 0) moves to (1, 3)



Note the domain is $\mathbb{R}$, the codomain is $\mathbb{R}$, and the range is $\mathbb{R}$.

Why are we bothering?
Notice for Calculus:

Math 171 Calculus I
Dr. McLoughlin's
Handy Dandy Guide to
Graphing Using Calculus

Consider f(x) to be a function in <u>simplified</u> form

Finding where f(x) is above or below the x - axis.
Prerequisite: High School Algebra/ Precalculus THIS IS ONLY A REVIEW.

set the numerator of f(x) = 0 yielding first coordinates of x - intercepts
set the denominator of f(x) = 0 yielding vertical asymptotes.

call these values *cut values.*

Do a Positive - negative Analysis of f(x) with the cut values

if f(x) > 0 , the the graph is above the x -axis for all values between the cut values
if f(x) < 0 , the the graph is below the x -axis for all values between the cut values

plug the cut values back into f(x). If it exists, it is the second coordinate of the x - intercept,
if it does not exist, then that cut value should have been the x - value of a vertical asymptote (or if you did not simplify the function, it could be a hole in the graph).

Finding where f(x) is increasing or decreasing.

First find f'(x)!

set the numerator of f'(x) = 0
set the denominator of f'(x) = 0 .

call these values *critical values.*

Do a Positive - negative Analysis of f'(x) with the critical values

if f'(x) > 0 , the the graph is increasing for all values between the critical values
if f'(x) < 0 , the the graph is decreasing for all values between the critical values

plug the critical value(s) back into f(x). If it exists, it is the second coordinate of a critical point, if it does not exist, then that critical value should have been the x - value of a vertical asymptote (or if you did not simplify the function, it could be a hole in the graph).

Any critical point where the function changed from increasing to decreasing is a **relative maxima.**
Any critical point where the function changed from decreasing to increasing is a **relative minima.**

Finding where f(x) is concave up or down.

First find $f''(x)$!

set the numerator of f''(x) = 0
set the denominator of f''(x) = 0 .

call these values *candidates for points of inflection (CPIs)*.

Do a Positive - negative Analysis of f''(x) with the CPIs

if f''(x) > 0 , the the graph is concave up for all values between the CPIs
if f''(x) < 0 , the the graph is concave down for all values between the CPIs

plug the CPI(s) back into f(x). If it exists, it is the second coordinate of a point,
if it does not exist, then that CPI value should have been the x - value of a vertical asymptote (or if you did not simplify the function, it could be a hole in the graph).

Any point that came from a CPI where the function changes concavity is **a point of inflection**.

Exercises:

1. Systematically graph $y = 3(x-2)^2 + 5$, ‘track’ the point (0, 0),
note the domain and the codomain before graphing, and the range after finishing.

2. Systematically graph $y = \frac{1}{3}(x+2)^4 + 6$, ‘track’ the point (0, 0),
note the domain and the codomain before graphing, and the range after finishing.

3. Systematically graph $y = \frac{4}{3}\sqrt[3]{x-2} + 5$, ‘track’ the point (0, 0),
note the domain and the codomain before graphing, and the range after finishing.

4. Systematically graph $y = \frac{3}{4}\sqrt{x-3} - 1$, 'track' the point (0, 0), note the domain and the codomain before graphing, and the range after finishing.

5. Systematically graph $y = (x+1)^{-1} + 2$ 'track' vertical and horizontal asymptotes, note the domain and the codomain before graphing, and the range after finishing.

6. Systematically graph $y = 2(x+1)^{1/4} + 3$, 'track' the point (0, 0), note the domain and the codomain before graphing, and the range after finishing.

7. Systematically graph $y = 4(x+1)^2 + 3$, 'track' the point (0, 0), note the domain and the codomain before graphing, and the range after finishing.

8. Systematically graph $y = \frac{1}{4}(x-1)^2 + 3$, 'track' the point (0, 0), note the domain and the codomain before graphing, and the range after finishing.

9. Systematically graph $y = -\frac{1}{4}(x+1)^2 - 3$, 'track' the point (0, 0), note the domain and the codomain before graphing, and the range after finishing.

End, last revised 9 October 2004.