## Basic Tests for Divisibility of Integers

<table>
<thead>
<tr>
<th>Factor</th>
<th>Test for Divisibility by Factor</th>
<th>Example(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Nothing is divisible by 0</td>
<td>None (cannot be done)</td>
</tr>
<tr>
<td>1</td>
<td>Everything is divisible by 1</td>
<td>For any integer $n$, the factors are 1 and $n$</td>
</tr>
<tr>
<td>2</td>
<td>$n$ must be even (i.e., the last digit of $n$ must be 0, 2, 4, 6, or 8)</td>
<td>Any even integer is divisible by 2</td>
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</tbody>
</table>
| 3      | Sum of digits of $n$ must be divisible by 3 | 1) $n = 42$  
6 is divisible by 3, so 42 is divisible by 3  
2) $n = 429,683,178$  
$4 + 2 + 9 + 6 + 8 + 3 + 1 + 7 + 8 = 48$  
$4 + 8 = 12$  
$1 + 2 = 3$  
3 is divisible by 3, so 429,683,178 is divisible by 3 |
| 4      | Last 2 digits of $n$ must be divisible by 4 | $n = 38,792$  
92 is divisible by 4 ($92 = 23 \times 4$), so 38,792 is divisible by 4 |
| 5      | Last digit of $n$ must be 5 or 0 | Any integer ending in 5 or 0 is divisible by 5 |
| 6      | $n$ must be divisible by both 2 and 3 | $n = 234$  
234 is even, so 234 is divisible by 2  
$2 + 3 + 4 = 9$, which is divisible by 3, so 234 is divisible by 3  
234 is divisible by both 2 and 3, so 234 is divisible by 6 |
| 7      | No easy test; see back of page for actual test | Guess and check is generally best approach |
| 8      | a) Last 3 digits of $n$ must be divisible by 8  
b) $n$ is divisible by 2, the result after dividing $n$ by 2 is divisible by 4  
c) $n$ is divisible by 4, the result after dividing $n$ by 4 is divisible by 2 | $n = 5824$  
a) 824 is divisible by 8, so 5824 is divisible by 8  
b) 5824 is even, so 5824 is divisible by 2  
$5824 + 2 = 2912$  
12 is divisible by 4, so 2912 is divisible by 4  
Therefore, 5824 is divisible by 8  
c) 24 is divisible by 4, so 5824 is divisible by 4  
$5824 + 4 = 1456$  
1456 is even, so 1456 is divisible by 2  
Therefore, 5824 is divisible by 8 |
| 9      | Sum of digits of $n$ must be divisible by 9 | 1) $n = 612$  
$6 + 1 + 2 = 9$  
9 is divisible by 9, so 612 is divisible by 9  
2) $n = 713,256,948$  
$7 + 1 + 3 + 2 + 5 + 6 + 9 + 4 + 8 = 45$  
$4 + 5 = 9$  
9 is divisible by 9, so 713,256,948 is divisible by 9 |
| 10     | Last digit of $n$ must be 0 | Any integer ending in 0 is divisible by 10 |
| 25     | Last 2 digits of $n$ must be divisible by 25  
(i.e., last 2 digits are 00, 25, 50, or 75) | Any integer ending in 00, 25, 50, or 75 is divisible by 25 |
| 100    | Last 2 digits of $n$ must both be 0 | Any integer ending in 00 is divisible by 100 |
Test for Divisibility of an Integer by 7

1. Double the last digit of the current value and subtract that result from the current value with the last digit omitted.

2. If the value obtained is divisible by 7, then the given number is divisible by 7; if the value obtained is not divisible by 7, then the given number is not divisible by 7.

3. Repeat the two preceding steps as many times as necessary until the number obtained can easily be seen to be divisible by 7 or not.

Examples:

a. 142,891

Solution:  
1) Double 1 to get 2 and subtract from 14,289 to get 14,287.  
2) Double 7 to get 14 and subtract from 1428 to get 1414.  
3) Double 4 to get 8 and subtract from 141 to get 133.  
4) Double 3 to get 6 and subtract from 13 to get 7.  
5) 142,891 is divisible by 7 since 7 is divisible by 7.

b. 409,311

Solution:  
1) Double 1 to get 2 and subtract from 40,931 to get 40,929.  
2) Double 9 to get 18 and subtract from 4092 to get 4074.  
3) Double 4 to get 8 and subtract from 407 to get 399.  
4) Double 9 to get 18 and subtract from 39 to get 21.  
(Note: We could stop at this point since 21 is divisible by 7.)  
5) Double 1 to get 2 and subtract from 2 to get 0.  
6) 409,311 is divisible by 7 since 0 is divisible by 7.

c. 458,485

Solution:  
1) Double 5 to get 10 and subtract from 45,848 to get 45,838.  
2) Double 8 to get 16 and subtract from 4583 to get 4567.  
3) Double 7 to get 14 and subtract from 456 to get 442.  
4) Double 2 to get 4 and subtract from 44 to get 40.  
(Note: We could stop at this point since 40 is not divisible by 7.)  
5) Double 0 to get 0 and subtract from 4 to get 4.  
6) 458,485 is not divisible by 7 since 4 is not divisible by 7.

d. 287,824

Solution:  
1) Double 4 to get 8 and subtract from 28,782 to get 28,774.  
2) Double 4 to get 8 and subtract from 2877 to get 2869.  
3) Double 9 to get 18 and subtract from 286 to get 268.  
4) Double 8 to get 16 and subtract from 26 to get 10.  
(Note: We could stop at this point since 10 is not divisible by 7.)  
5) Double 0 to get 0 and subtract from 1 to get 1.  
6) 287,824 is not divisible by 7 since 1 is not divisible by 7.